MATH 115 Sections 5.1a
Introduction

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1. **Introduction:** The proper definitions of the trigonometric functions are based on the *unit circle* and some associated definitions, so we'll examine that first.

2. **The Unit Circle:** The *unit circle* (henceforth abbreviated UC) is the circle of radius 1 centered at the origin. Note that its equation is $x^2 + y^2 = 1$.

Example: Is the point $(0.2, 0.8)$ on the unit circle? Since $(0.2)^2 + (0.8)^2 \neq 1$, we know it's not.

Example: Find all x so that $\left(x, \frac{\sqrt{2}}{2}\right)$ is on the unit circle. Suppose the point is in the fourth quadrant. Then what?

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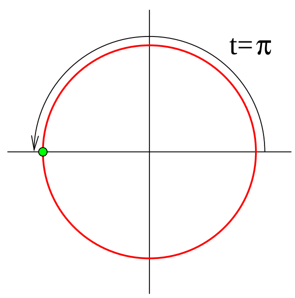
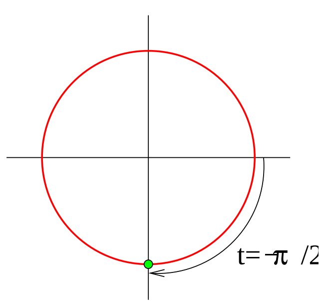
3. **Terminal Points:** Suppose t is a number. Imagine you are at $(1, 0)$ on the UC and you walk counterclockwise if $t > 0$ and clockwise if $t < 0$. You go a distance of $|t|$ as measured along the UC itself. Where are you? Clearly this depends upon t. The point you end up at is called the *terminal point for* t.

Example: Find the terminal point for $t = \pi$. Note that the UC has circumference 2π (because it's $2\pi r$) and so a trip all the way around is distance 2π. A distance of π is then a trip halfway around:



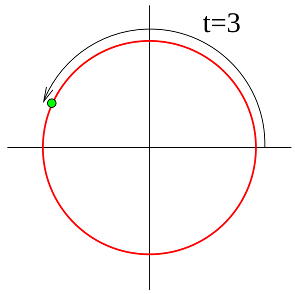
We end up at $(-1, 0)$ and so the terminal point for $t = \pi$ is $(-1, 0)$.

Example: Find the terminal points for $t = -\frac{\pi}{2}$. Since $t < 0$ we go clockwise by $\frac{\pi}{2}$. This is a fourth of the way around (a fourth of 2π).



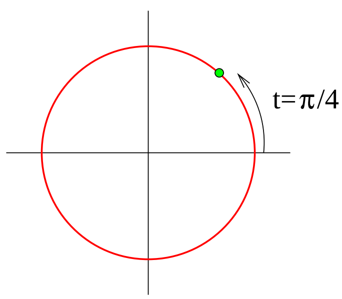
We end up at $(0, -1)$ and so the terminal point for $t = -\frac{\pi}{2}$ is $(0, -1)$.

It's not always obvious where the terminal point is. For example, where is the terminal point for $t = 3$? Since 3 is slightly less than π and since the terminal point for π is halfway around, the terminal point for 3 should be slightly less. It's right about here:



But where is it? Looks like, approximately, $(-0.9, 0.2)$ or so. This is just an approximation.

There are several terminal points we need to know on sight. One is the terminal point for $\frac{\pi}{4}$. Note that this is located here:



So where is this? Well, notice that by symmetry $x = y$ and since the point is on the UC $x^2 + y^2 = 1$. Thus $x^2 + x^2 = 1$ and so $x = \pm\frac{\sqrt{2}}{2}$. Since it's in the first quadrant it's $x = \frac{\sqrt{2}}{2}$ and then $y = \frac{\sqrt{2}}{2}$ also. Therefore the terminal point for $t = \frac{\pi}{4}$ is $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

A similar argument can show us that the terminal point for $t = \frac{\pi}{6}$ is $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and the terminal point for $t = \frac{\pi}{3}$ is $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

In summary for now on the UC we have the following five very important points: