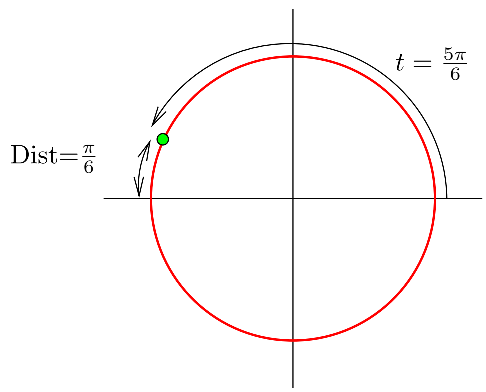
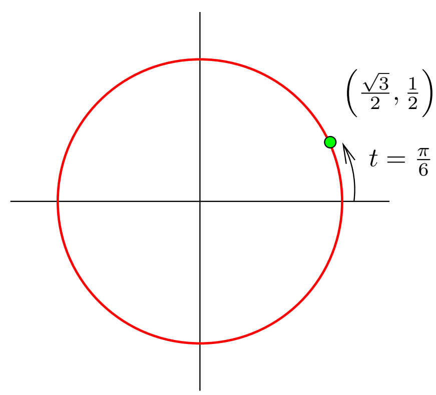
MATH 115 Sections 5.1b/5.2a

1. **A Comment About Reference Numbers:** On Monday we looked at some basic terminal points and now we're going to purposefully complicate things a bit. The idea of using reference numbers is a formal way of doing what we do in our heads and in truth it will be obsolete soon. Until then it's good for working up some fluency with the unit circle.

2. **Pre-Definition:** Suppose we asked for the terminal point for $t = \frac{5\pi}{6}$. Probably what you would do is this: First you note that $\frac{5\pi}{6}$ is $\frac{\pi}{6}$ less than half a rotation (π, or $\frac{6\pi}{6}$):



then you recall from Monday that $\frac{\pi}{6}$ itself has terminal point located here:



and you note there's some symmetry there. We see that the point we want in the first picture must be $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ because it's in the second quadrant.

What we've done here is intuitively define the reference number.

Definition: For some t, the *reference number* for t is the distance (along the UC) from the terminal point for t to the x-axis. To find this we need to know where the terminal point for t is located and do a quick calculation.

Example: The reference number for $t = \frac{5\pi}{6}$ is $\frac{\pi}{6}$.

Example: Find the reference number for each of the following. Note that a small UC with a ballpark approximation of where the TP is located is necessary for most of these: $t = -\frac{\pi}{4}$, $t = -\frac{11\pi}{6}$, $t = \frac{19\pi}{12}$, $t = \frac{\pi}{3}$, $t = 3$ and $t = 6$.

Using the Reference Number: We can now write down a formal way to use the reference number to find the terminal point:

(a) For any t, figure out which quadrant the TP is in.

(b) Then find the reference number.

(c) Find the TP for the reference number.

(d) Adjust the $\pm$ for each coordinate depending upon where the TP for the original value is.

Example: Use the reference number to find the TP for $t = \frac{11\pi}{6}$.

Example: Use the reference number to find the TP for $t = \frac{5\pi}{3}$.

Example: Use the reference number to find the TP for $t = \frac{3\pi}{4}$.

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3. **Definition of the Trigonometric Functions:** We are now in a position to defind the trigonometric functions. Suppose t is a number. Suppose (x, y) is the TP for t. Then we define

The sine of t:	$\sin(t) = y$
The cosine of t:	$\cos(t) = x$
The tangent of t:	$\tan(t) = \frac{y}{x}$
The cotangent of t:	$\cot(t) = \frac{x}{y}$
The secant of t:	$\sec(t) = \frac{1}{x}$
The cosecant of t:	$\csc(t) = \frac{1}{y}$

Now we can actually compute some trig functions from the very raw definitions:

Example: Find $\sin\left(\frac{\pi}{6}\right)$. First note that $\frac{\pi}{6}$ has terminal point $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. Since sine is the $y-$value, the answer is $\frac{1}{2}$. That is, $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.

Example: From this point we can find all the trigonometric functions at $\frac{\pi}{6}$. Find them.

Example: Find the following: $\tan(0)$, $\csc(\pi)$, $\cos\left(\frac{5\pi}{4}\right)$ and $\cot\left(\frac{2\pi}{3}\right)$.
