MATH 115 Sections 5.2b Lecture Notes

1. **Reminder:** We've defined the trigonometric functions for any t. Today we'll do two things. First, list some trig. functions you need to know and second list some properties of the functions which follow from the definitions. You need to memorize all of these.

2. **Reciprocal Identities:** The *reciprocal identities* are the most useful identities of the trigonometric functions. These follow straight from the definitions. The first five are the most useful but keep the last three in mind too.

(a) $\tan t = \frac{\sin t}{\cos t}$	(e) $\tan t = \frac{1}{\cot t}$
(b) $\cot t = \frac{\cos t}{\sin t}$	(f) $\sin t = \frac{1}{\csc t}$
(c) $\sec t = \frac{1}{\cos t}$	(g) $\cos t = \frac{1}{\sec t}$
(d) $\csc t = \frac{1}{\sin t}$	(h) $\cot t = \frac{1}{\tan t}$

   Observe that knowing $\sin t$ and $\cos t$ gives us everything else, but often it's best to know $\tan t$ without calculation.

3. **Know These:** It's a good idea to memorize all of the following:

t	$\sin t$	$\cos t$	$\tan t$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	Und

   As mentioned before, tan can be derived from sin and cos but it comes up so frequently that you ought to know it.

4. **Sign Identities** Since the trig functions are based upon the x and y values of the terminal points we can easily figure out whether they are positive or negative depending on where the terminal points for t is. For example if the TP for t is in Q2 then $\cos(t) < 0$ (the $x-$value) and $\sin(t) > 0$ (the $y-$value). In summary just remember for sin, cos and tan and the other three follow by reciprocals. The rule is best remembered by the mnemonic picture:

S	A
T	C

   The letters indicate which of the big three are positive. A=all, S=sine, T=tangent and C=cosine. The rest of the big three are negative. Remember that All Students Take Calculus.

   Example: Is $\cos(4)$ positive or negative? How about $\tan(4)$?

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5. **Even-Odd Properties:** Observe that the TP for t and $-t$ are symmetric with respect to the $x-$axis. Consequentially their $x-$values (cosines) are the same. Their $y-$values (sines) are opposite though, as are their tangents. In summary:

   (a) $\sin(-t) = -\sin(t)$
   (b) $\cos(-t) = \cos(t)$
   (c) $\tan(-t) = -\tan(t)$

   Example: $\sin\left(-\frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$.

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6. **Calculating:** We can now calculate the trigonometric functions of any nice t, where "nice" means that the reference number $\bar{t}$ is one of our five good values. Keep in mind that we can move from t to the reference number $\bar{t}$ provided we adjust the $\pm$ for the quadrant.

Example: Find $\cos\left(-\frac{4\pi}{3}\right)$.

Solution: By the even-odd property we can ditch the $-$ sign. The TP for $t = \frac{4\pi}{3}$ is in Q3 and the reference number is $\frac{\pi}{3}$. In Q3 we know cosine is negative and so in summary:

$$\cos\left(-\frac{4\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$$

Example: Find $\cot\left(\frac{7\pi}{4}\right)$.

Solution: Cotangent is the reciprocal of tangent. The TP for $t = \frac{7\pi}{4}$ is in Q4 and the reference number is $\frac{\pi}{4}$. In Q4 we know tangent is negative and so in summary:

$$\cot\left(\frac{7\pi}{4}\right) = \frac{1}{\tan\left(\frac{7\pi}{4}\right)} = -\frac{1}{\tan\left(\frac{\pi}{4}\right)} = -\frac{1}{1} = -1$$

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7. **Pythagorean Identities:** Lastly there are three identities that will be amongst the most useful both here and especially in calculus. Note that on the unit circle we have $x^2 + y^2 = 1$ and since $x = \cos(t)$ and $y = \sin(t)$ we have $(\cos(t))^2 + (\sin(t))^2 = 1$. But a notational issue first: For trig functions, rather than writing $(\sin(t))^2$ we write $\sin^2(t)$, and usually here sine is listed first. Thus we have

   (a) $\sin^2 t + \cos^2 t = 1$

   We also have two friends of this identity:

   (b) $1 + \tan^2 t = \sec^2 t$

   (c) $1 + \cot^2 t = \csc^2 t$

   The last two are derived from the first one by dividing through by $\cos^2 t$ and $\sin^2 t$ respectively.

8. **Final Problem Type:** Here is a problem which brings together much of what we've seen today:

Example: Suppose $\sin t = \frac{1}{4}$ and (the TP for) t is in Q2. Find $\cos t$, $\tan t$ and $\sin(t + \pi)$.

Solution: By the Pythagorean identity $\left(\frac{1}{4}\right)^2 + \cos^2 t = 1$ and since we're in Q2 cosine is negative so $\cos t = -\frac{\sqrt{15}}{4}$. Then $\tan t = \frac{\sin t}{\cos t} = -\frac{1}{\sqrt{15}}$. Note then that (the TP for) $t+\pi$ is across the UC from (the TP for) t and so the x and y (hence sin and cos) are negated. Thus $\sin(t+\pi) = -\frac{1}{4}$.

Example: Suppose $\tan t = 0.6$ and (the TP for) t is in Q3. Find $\sec t$, $\cos t$ and $\sin t$.

Solution: Find $\sec t$ by the Pythagorean Identity then $\cos t$ by reciprocal and then $\sin t$ by $\tan t = \frac{\sin t}{\cos t}$ (or by the other Pythagorean Identity).
