# MATH 115 Sections 5.3a Lecture Notes

1. **Introduction:** This section addresses both the graphs of sine and cosine. Today we'll do sine. Cosine is very close and can be done quickly once sine is known.

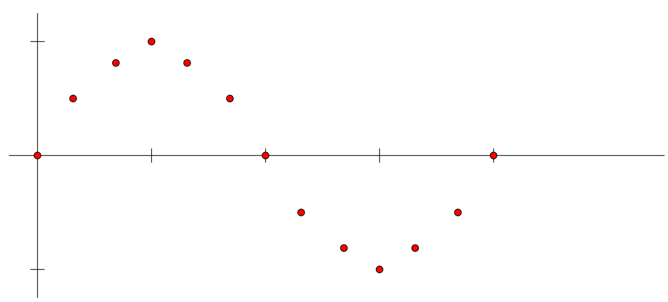
2. **A Note on Sine as a Function:** Up until now we've been thinking of $\sin(t)$ as a function of $t$. In this context we thought of it as the $y-$value of the terminal point of $t$. But functions are traditionally given as functions of $x$. Thus we'll start writing $\sin(x)$ and try to get away from the connection to the unit circle even though the definition is in the background. In other words $\sin(x)$ is the $y-$value of the terminal point of $x$.

3. **The Graph of Sine:** Before plotting any points, note that the terminal point for $x + 2\pi$ is the same as the terminal point for $x$. This means that $\sin(x + 2\pi) = \sin(x)$. What this means is that sine repeats every $2\pi$. Formally we say that sine is *periodic with period* $2\pi$. To graph the function then we only need to sketch it between $x = 0$ and $x = 2\pi$ and then it repeats over and over.

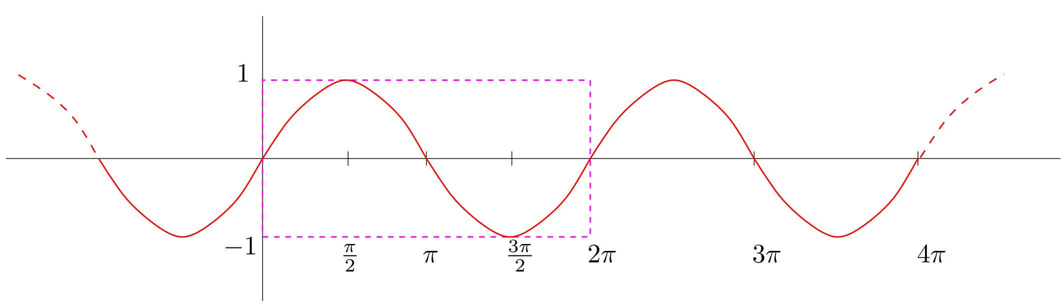
   Given this fact we can create a list of points for the graph of $\sin(x)$ by plugging in various values for $x$. We won't list all the $x$ here, just enough to draw the graph:

| $x$ | $0$ | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{5\pi}{6}$ | $\pi$ | $\frac{7\pi}{6}$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ | $\frac{5\pi}{3}$ | $\frac{11\pi}{6}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sin(x)$ | $0$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $1$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $0$ | $-\frac{1}{2}$ | $-\frac{\sqrt{3}}{2}$ | $-1$ | $-\frac{\sqrt{3}}{2}$ | $-\frac{1}{2}$ | $0$ |

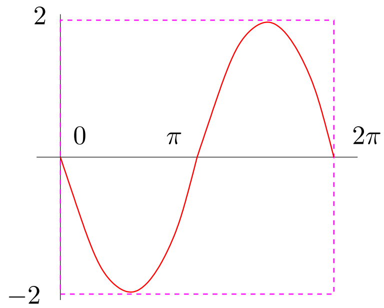
   Now if we plot these as points on a graph we get the following:



   If we fill these in with a curve and repeat it over and over we get the following picture. For future reference I have boxed in a single *period* of the graph:



4. **Amplitude and Reflection Variations:** Consider the function $f(x) = -2\sin(x)$. We know from our study of transformations that the 2 stretches the graph vertically by a factor of 2 and the $-$ reflects it in the $x-$axis. We will draw just a single period of this:

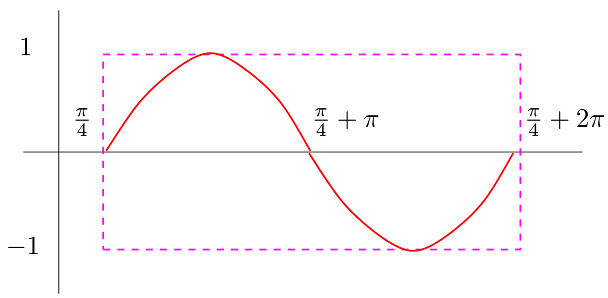



Again note that I've drawn in the box containing a single period. This will be useful later.

In general the graph of $f(x) = a\sin(x)$ could involve a reflection and a stretch or shrink.

Definition: The value $|a|$ is the *amplitude* of the function.

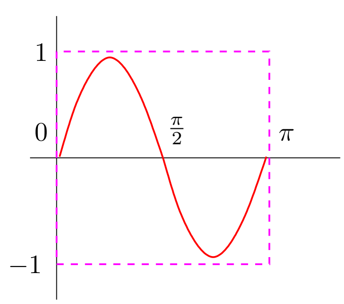
5. **Phase Shift:** Consider the function $f(x) = \sin\left(x - \frac{\pi}{4}\right)$. We know that this is a shift to the right by $\frac{\pi}{4}$. Here is the graph of a single period:




In general the graph of $f(x) = \sin(x - b)$ repositions the start of the period at $x = b$.

Definition: The value of $b$ is the *phase shift* of the function.

6. **Period Variations:** We never spent any time on horizontal stretching but we need it here. Here is an example to help us in general. Consider $f(x) = \sin(2x)$. The 2 has the effect of shrinking horizontally by a factor of 2. The period was $2\pi$ and now it's $\frac{2\pi}{2} = \pi$. Thus the graph is:




In general the *period* of $f(x) = \sin(kx)$ is $\frac{2\pi}{k}$.

7. **All Together:** Putting it all together we look at the function $f(x) = a \sin k(x - b)$. We have:

   Amplitude $= |a|$

   Phase shift $=$ Start of period $= b$

   Period $= \frac{2\pi}{k}$

   In order to draw the graph, it's very helpful to first simply draw a box which fits the criteria above and then jam a single period of sine inside it, flipped if necessary.

---

Example: Sketch $f(x) = 3\sin\left(\frac{1}{2}x + \frac{\pi}{12}\right)$.

First rewrite this in the correct form as: $f(x) = 3\sin\frac{1}{2}\left(x - \left(-\frac{\pi}{6}\right)\right)$.

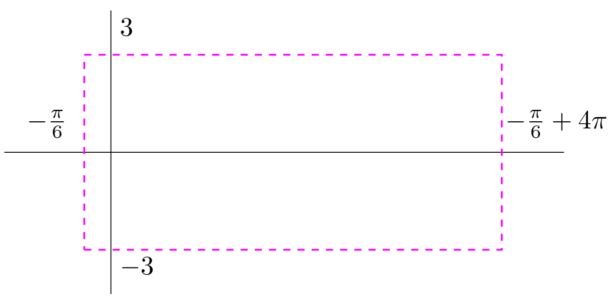
And then note that we have:

Amplitude$=|3| = 3$ with no flip.

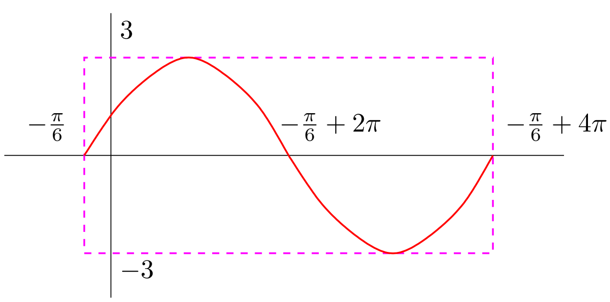
Phase shift $=$ start of period $= -\frac{\pi}{6}$.

Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$.

Thus our one-period box looks like:




and filled in with a period:




---