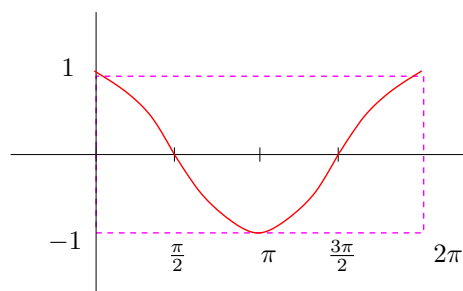
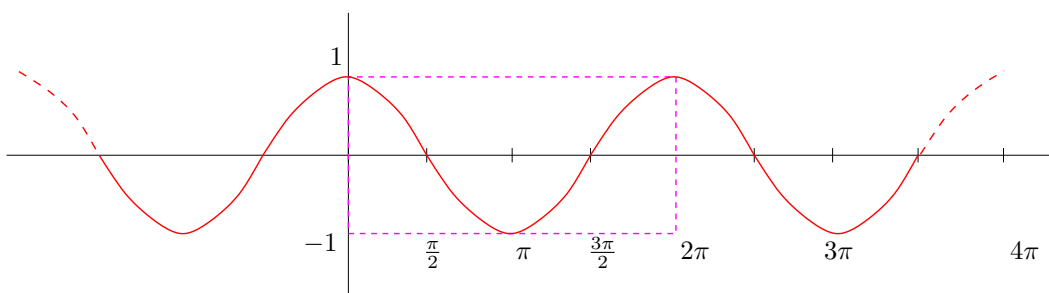


MATH 115 Sections 5.3b Lecture Notes

1. **Introduction:** Now that we know how to draw sine, drawing cosine is actually very easy.
2. **The Basic Period of Sine:** Cosine, like sine, repeats every 2π units. If we plot points (which we won't) we find out that the basic period, between $x = 0$ and $x = 2\pi$, looks like this:



Hence if we repeat it over and over we get the following.



3. **General Cosine:** Other than this slight difference in the basic period, cosine works exactly like sine. That is, for $f(x) = a \cos k(x - b)$ we have:

$$\text{Amplitude} = |a|$$

$$\text{Phase shift} = \text{Start of period} = b$$

$$\text{Period} = \frac{2\pi}{k}$$

So then just like sine, first simply draw a box which fits the criteria above and then jam a single period of cosine inside it, flipped if necessary.

In other words, graphing sine and cosine involve exactly the same process up until you fill the curve in the box.

4. **Examples:** Here are a few examples, the first with more detail than the others:

Example: Sketch $f(x) = 3 \cos\left(\frac{1}{2}x + \frac{\pi}{12}\right)$.

First rewrite this in the correct form as: $f(x) = 3 \cos\frac{1}{2}\left(x - \left(-\frac{\pi}{6}\right)\right)$.

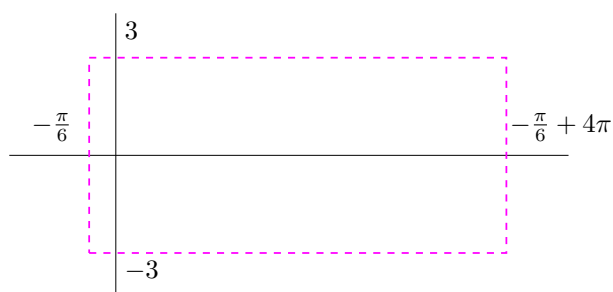
And then note that we have:

Amplitude= $|3| = 3$ with no flip.

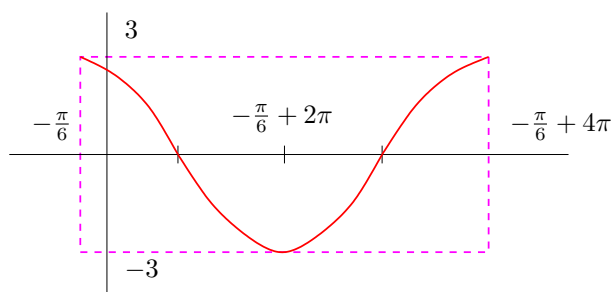
Phase shift = start of period = $-\frac{\pi}{6}$.

Period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$.

Thus our one-period box looks like:



and filled in with a period:



Note that the halfway mark is easy to see but unlike sine it's not an intercept. Here the x -intercepts are harder to see, but here is a way to think of them:

The first is $\frac{1}{4}$ th of the way through the period, so it's at $-\frac{\pi}{6} + \frac{1}{4}(4\pi)$.

The second is $\frac{3}{4}$ th of the way through the period, so it's at $-\frac{\pi}{6} + \frac{3}{4}(4\pi)$.

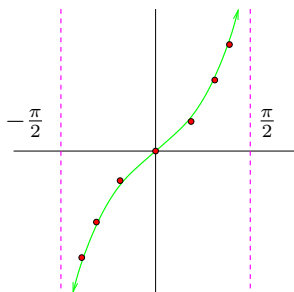
Example: Sketch $f(x) = -1.4 \cos\frac{\pi}{2}(x - 1)$.

This is sneaky because the period is $\frac{2\pi}{(\frac{\pi}{2})} = 4$, a nice round number with no π in it, and the phase shift is $b = 1$.

5. **Graph of Tangent:** The graph of tangent is completely unlike that of sine and cosine for two really big reasons. One is that it repeats every π units rather than every 2π units. Two is that it is undefined in some places. To plot some points then, instead of starting at $x = 0$ we'll start at $x = -\frac{\pi}{2}$. The reason for this will become clear:

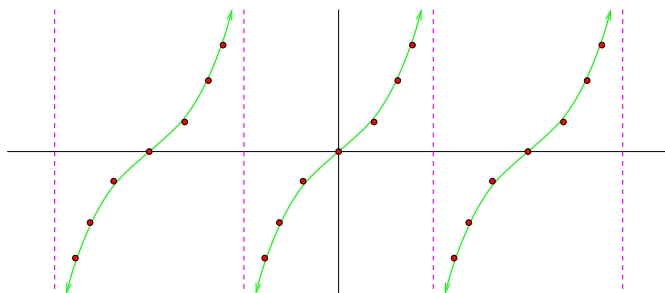
x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\tan(x)$	Undef	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undef

If we plot these and fill them in, we see the following:



Observe that there are vertical asymptotes at $\pm\frac{\pi}{2}$. This is since $\tan(x) = \frac{\sin(x)}{\cos(x)}$ and these occur where $\cos(x)$ go to 0 so the fraction goes to $\pm\infty$.

Since $\tan(x)$ repeats every π , what we really get is the following picture:



6. **Variations on Tangent:** Tangent may be modified much like sine and cosine but we need to be careful about the fact that tangent starts with a shorter period and we draw a different domain of values for the standard period.

For $f(x) = a \tan k(x - b)$ we have:

$$\text{Amplitude} = |a|$$

$$\text{Phase shift} = \text{Middle of period} = b$$

$$\text{Period} = \frac{\pi}{k}$$

$$\text{Asymptotes are at } b \pm \frac{1}{2}(\text{Period}).$$

Note the differences. Amplitude is still called as such even though it no longer indicates the height as with sine and cosine. The phase shift now indicates the middle of the period while the period itself starts with π and gets adjusted according to k .

The easiest way to draw tangent is to figure out the information and draw the asymptotes. Then fill in one period of tangent, flipped if necessary.

Example: Sketch $f(x) = 2 \tan \left(2x - \frac{\pi}{3}\right)$.

First we rewrite this as $f(x) = 2 \tan 2 \left(x - \frac{\pi}{6}\right)$ and then note:

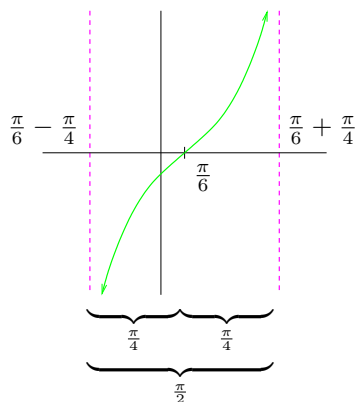
$$\text{Amplitude} = |2| = 2, \text{ which is meaningless except as a stretch.}$$

$$\text{Phase shift} = \text{Middle of period} = \frac{\pi}{6}.$$

$$\text{Period} = \frac{\pi}{2}.$$

$$\text{Asymptotes are at } \frac{\pi}{6} \pm \frac{1}{2} \left(\frac{\pi}{2}\right)$$

Thus we have:



Note that I put some braces in to make sure the scale of things is more clear. These are of course not necessary.