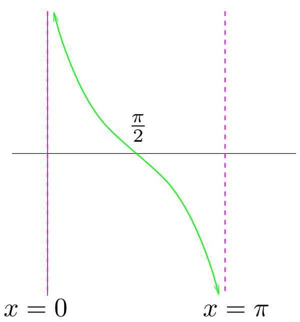
MATH 115 Section 5.4 Lecture Notes

1. **Introduction:** Now that we know what sine, cosine and tangent look like we can use them to understand what cosecant, secant and cotangent look like. We won't go through too much detail, we'll just summarize:

2. **Summary of Cotangent:** Cotangent has the basic shape:



In general for $f(x) = a \cot k(x - b)$ we have:

Amplitude=$|a|$ with a flip if $a < 0$.

Phase shift=b=start of period.

Period=$\frac{\pi}{k}$.

Asymptotes at $x = b$ and $x = b$+period.

Example: $f(x) = \frac{1}{2} \cot 3 \left(x - \frac{\pi}{7}\right)$

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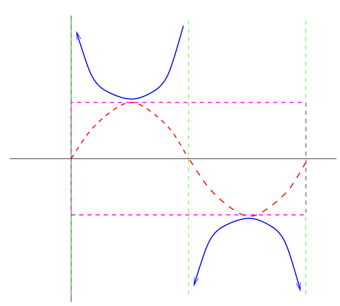
3. **Cosecant:** We'll analyze $\csc x = \frac{1}{\sin x}$ and then just make the parallel observation for $\sec x = \frac{1}{\cos x}$. Think about what happens if you take the reciprocal of a graph. In other words, you take the reciprocal of the $y-$values. If $y = 1$ then the reciprocal is 1. If $y > 1$ then the reciprocal is < 1 and if $y < 1$ the reciprocal is > 1. This assumes $y > 0$ but we can see more clearly with a picture. Here is a period of $\sin x$ along with its reciprocal, $\csc x$:



Note: Once you have the graph you can describe in more detail what csc is doing as sin changes. This is much easier with the graph available.

Note that when $\sin x = 0$ we have $\csc x$ undefined and there are vertical asymptotes there.

The easiest way to sketch $\csc x$ is then to lightly sketch $\sin x$ and then take the visual reciprocal. In other words, draw in the associated curves as we see here.

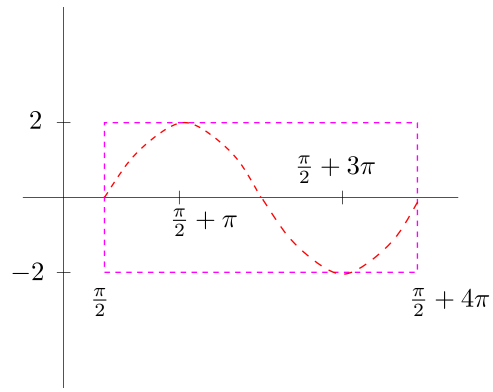
Example: Sketch $f(x) = 2\csc\frac{1}{2}\left(x - \frac{\pi}{2}\right)$

What we do is simply draw $2\sin\frac{1}{2}\left(x - \frac{\pi}{2}\right)$ lightly and then draw csc using it as a guide:

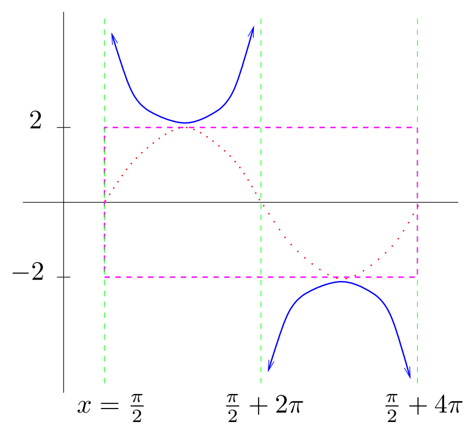
We have:

> Amplitude$=|2| = 2$ with no flip.
>
> Phase shift$=\frac{\pi}{2}$=start of period.
>
> Period$=\frac{2\pi}{\left(\frac{1}{2}\right)} = 4\pi$

So if we drew sine we'd have:



and if we now overlay a period of cosecant we'd have:



Example: Sketch $f(x) = -\csc\left(2x + \frac{\pi}{5}\right)$.

4. **Secant:** Secant is the reciprocal of cosine and so what we'll do is draw cosine and then "reciprocal" it to get the secant. This leaves asymptotes in weird places but so be it; it's good enough for now.

Example: Sketch $f(x) = \frac{1}{3}\sec\left(2x - \frac{\pi}{4}\right)$.

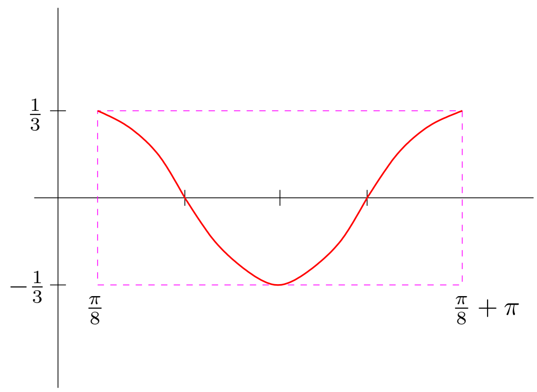
First we'll do cosine. We rewrite as $f(x) = \frac{1}{3}\sec 2\left(x - \frac{\pi}{8}\right)$ and then we have:

Amplitude$=|\frac{1}{3}| = \frac{1}{3}$.

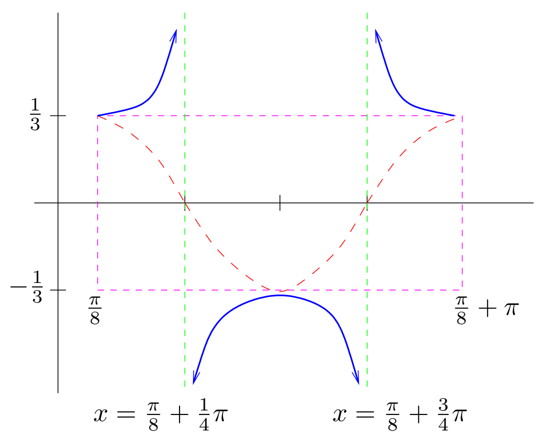
Phase shift$=\frac{\pi}{8}$=start of period.

Period$=\frac{2\pi}{2} = \pi$.

So if we drew cosine we'd have:



and if we now overlay a period of secant we'd have:



Example: Sketch $f(x) = -2\sec 0.2\left(x + \frac{\pi}{4}\right)$.
