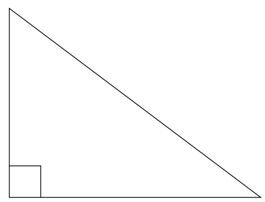
MATH 115 Section 6.2,6.3 Lecture Notes

1. **Introduction:** Note that section 6.3 is essentially a repeat of some of the material covered in 5.2. The reason it's done again here is that some courses (not at UMD) which use this book do not cover chapter 5 and so it's here to be complete. I generally make a comment to this effect to the students and then essentially skip the entire section because they know it already.

2. **Right Triangles:** A *right angle* is an angle which measures $90° = \frac{\pi}{2}$. A *right triangle* is a triangle in which one of the angles is right. This is indicated by a little box in the corner of the triangle:



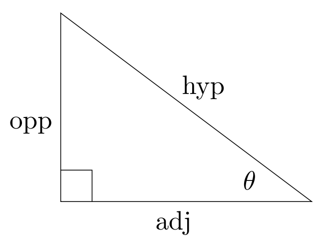
   There are two other angles which are not right. Even though there is no sense of initial or terminal side here, we take it that the other angles are between $0°$ and $90°$.

   The two sides which meet at the right angle are the *legs* and the long side is the *hypoteneuse*.

   Note that the Pythagorean Theorem tells us that the sum of the squares of the legs equals the square of the hypoteneuse. Or $\text{hyp}^2 = (\text{leg}\#1)^2 + (\text{leg}\#2)^2$.

   Also note that the sum of the three angles of any triangle add to $180° = \pi$, so the non-right angles of a right triangle add to $90° = \frac{\pi}{2}$.

3. **New Definitions of the Trigonometric Functions:** For each of these other two angles we can define six trigonometric functions as follows. Suppose θ is one of the angles. Let "opp" denote the length of the opposite leg, "adj" the adjacent leg and "hyp" the hypoteneuse.

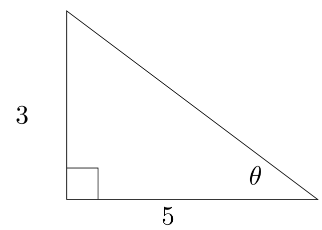


   Then we can define the following:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{\text{hpy}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

   A good mnemonic for the first three is `SOHCAHTOA` (sine is opp over hyp and so on) and then the other three are the reciprocals of the first three.

Example: Find $\sin\theta$ and $\tan\theta$ for the triangle shown:

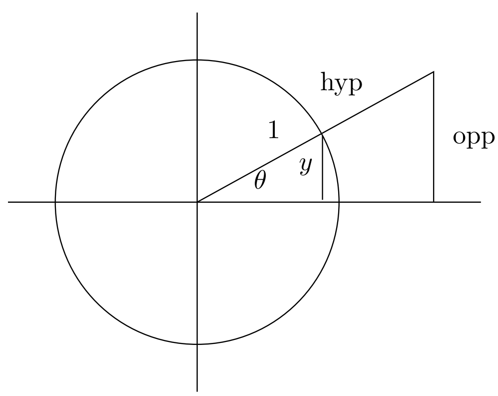


4. **Important Note:** It is important to realize something here. These are at first glance brand new definitions of the trigonometric functions. If I were to ask you for $\sin\frac{\pi}{6}$, for example, what do you do?

   (a) Old way: Find the y-coordinate of the terminal point for $\frac{\pi}{6}$?

   (b) New way: Construct a right triangle with angle $\frac{\pi}{6}$ and then do $\frac{\text{opp}}{\text{hyp}}$?

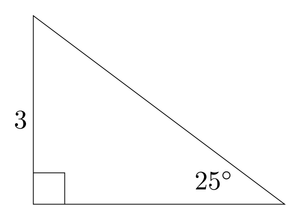
   Does it matter? If it did we'd have contradicting definitions and that would be irresponsible. It turns out that it doesn't matter. For a simple example, look at this picture:



   Using the new way, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$ but by similar triangles this equals $\frac{y}{1}$ which equals y, which is the same as the old way!

5. **Completing a Triangle:** To complete a triangle means to fill in all the angles and sides. This is very easy for right triangles.

Example: Complete the triangle shown here. Give both exact and approximate answers. Simplify the trig as much as possible.



6. **Applications:** Applications abound!

Example: A ladder leans against a wall and makes an angle of 60° with the ground. If the ladder is 20 feet long, how far up the wall does it reach?

Example: An eagle soaring at a height of 1000 feet sees a mouse at an angle of depression of 10°. How far is the eagle from the mouse?
