1. **Introduction:** Note that section 6.3 is essentially a repeat of some of the material covered in 5.2. The reason it’s done again here is that some courses (not at UMD) which use this book do not cover chapter 5 and so it’s here to be complete. I generally make a comment to this effect to the students and then essentially skip the entire section because they know it already.

2. **Right Triangles:** A right angle is an angle which measures \(90^\circ = \frac{\pi}{2}\). A right triangle is a triangle in which one of the angles is right. This is indicated by a little box in the corner of the triangle:

![Right Triangle Diagram]

There are two other angles which are not right. Even though there is no sense of initial or terminal side here, we take it that the other angles are between \(0^\circ\) and \(90^\circ\). The two sides which meet at the right angle are the legs and the long side is the hypoteneuse. Note that the Pythagorean Theorem tells us that the sum of the squares of the legs equals the square of the hypoteneuse. Or \(\text{hyp}^2 = (\text{leg#1})^2 + (\text{leg#2})^2\).

Also note that the sum of the three angles of any triangle add to \(180^\circ = \pi\), so the non-right angles of a right triangle add to \(90^\circ = \frac{\pi}{2}\).

3. **New Definitions of the Trigonometric Functions:** For each of these other two angles we can define six trigonometric functions as follows. Suppose \(\theta\) is one of the angles. Let “opp” denote the length of the opposite leg, “adj” the adjacent leg and “hyp” the hypoteneuse.

![Trig Functions Diagram]

Then we can define the following:

\[
\begin{align*}
\sin \theta &= \frac{\text{opp}}{\text{hyp}} & \cos \theta &= \frac{\text{adj}}{\text{hyp}} & \tan \theta &= \frac{\text{opp}}{\text{adj}} \\
\csc \theta &= \frac{\text{hyp}}{\text{opp}} & \sec \theta &= \frac{\text{hyp}}{\text{adj}} & \cot \theta &= \frac{\text{adj}}{\text{opp}}
\end{align*}
\]

A good mnemonic for the first three is **SOHCAHTOA** (sine is opp over hyp and so on) and then the other three are the reciprocals of the first three.
Example: Find \( \sin \theta \) and \( \tan \theta \) for the triangle shown:

\[
\begin{array}{c}
3 \\
\theta \\
5
\end{array}
\]

4. **Important Note:** It is important to realize something here. These are at first glance brand new definitions of the trigonometric functions. If I were to ask you for \( \sin \frac{\pi}{6} \), for example, what do you do?

(a) Old way: Find the \( y \)-coordinate of the terminal point for \( \frac{\pi}{6} \)?

(b) New way: Construct a right triangle with angle \( \frac{\pi}{6} \) and then do \( \frac{\text{opp}}{\text{hyp}} \)?

Does it matter? If it did we’d have contradicting definitions and that would be irresponsible. It turns out that it doesn’t matter. For a simple example, look at this picture:

Using the new way, \( \sin \theta = \frac{\text{opp}}{\text{hyp}} \) but by similar triangles this equals \( \frac{y}{1} \) which equals \( y \), which is the same as the old way!

5. **Completing a Triangle:** To complete a triangle means to fill in all the angles and sides. This is very easy for right triangles.

Example: Complete the triangle shown here. Give both exact and approximate answers. Simplify the trig as much as possible.

\[
\begin{array}{c}
3 \\
25^\circ
\end{array}
\]
6. **Applications**: Applications abound!

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Example: A ladder leans against a wall and makes an angle of 60° with the ground. If the ladder is 20 feet long, how far up the wall does it reach?

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Example: An eagle soaring at a height of 1000 feet sees a mouse at an angle of depression of 10°. How far is the eagle from the mouse?