MATH 115 Section 6.4 Lecture Notes

1. **Introduction:** The trigonometric functions of right triangles along with the Pythagorean Theorem and the fact that the angles of a triangle add to 180° allow us to find any other measurement of the triangle. But what if the triangle is not a right triangle? All of a sudden the Pythagorean Theorem does not apply and all that $\sin\theta = \frac{\text{opp}}{\text{hyp}}$ stuff fails as well.

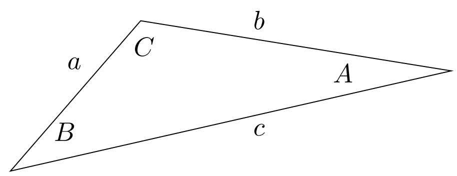
   In this case we need to use two new laws. One is the Law of Sines and the other is the Law of Cosines. The Law of Sines is harder - we will do it today. The Law of Cosines will be done on Monday.

2. **Cases:** Before we give the law we need to understand what combinations of information may be given. By this we mean that something must be given - some sides or angles. There are five possibilities with codes as follows:

   (a) ASA case: We're given two angles and a side between.
   (b) SAA case: We're given two angles and a side not between.
   (c) SSA case: We're given two sides and an angle not between.
   (d) SAS case: We're given two sides and an angle between.
   (e) SSS case: We're given three sides.

   The Law of Sines will take care of ASA, SAA and SSA and the Law of Cosines will take care of SAS and SSS.

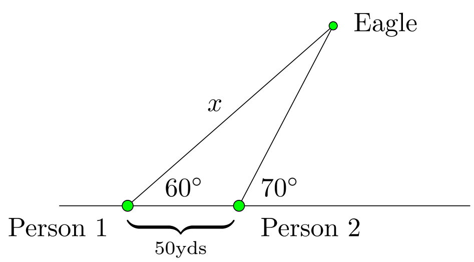
3. **Law of Sines:** Suppose we have a triangle with sides a, b, c and angles A, B, C, where the angles are opposite their corresponding sides:



   Then the Law of Sines says that

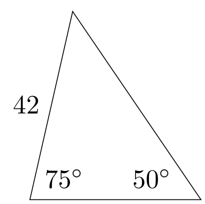
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

4. **ASA Case, Application:** Two people are standing 50 yards apart on a road. They both look to the right and see the same eagle and they measure the angles as shown. How far from the first person is the eagle?



   Note that the missing angles are 110° and 10° and hence $\frac{\sin 110^\circ}{x} = \frac{\sin 10^\circ}{50}$. Then $x = \frac{50 \sin 110^\circ}{\sin 10^\circ}$

5. **SAA Case, Example:** Solve the triangle shown below:



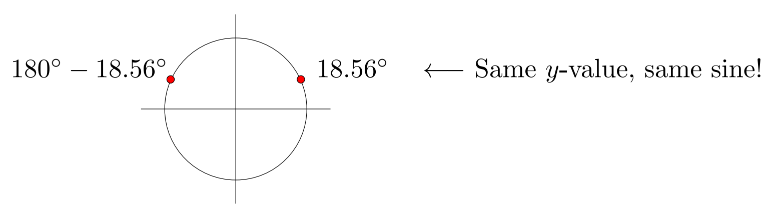
6. **SSA Case, Icky:** Things run pretty smoothly until we get to the SSA case. The reason is this: It's possible, given SSA, to have either one solution, two solutions or no solutions. This may seem worrisome but in truth we generally charge along and the situation reveals itself to us as we go. In other words we won't know ahead of time what's up but we'll find it as we go.

   The other reason is that SSA problems involve solving things like $\sin A = \frac{1}{2}$. While we can do this particular example ($A = 30°$ or $A = 150°$), what if we get $\sin A = 0.6$? Welcome to the $\sin^{-1}$ button on your calculator. This is the inverse sine and does this:

   Magic but Stupid: $\sin^{-1} 0.6$ will give you the angle whose sine is 0.6. It will use whatever units (radians or degrees) your calculator is set for. Since there are many answers and the calculator is stupid, it gives you the angle between 0 and 90°. Rather than wade too deep, let's see how this comes out with some examples. The first will not need the inverse sine button.

   (a) Suppose a triangle has $A = 45°$, $a = 7\sqrt{2}$ and $b = 7$. Find B.
   By the LOS, $\frac{\sin B}{7} = \frac{\sin 45°}{7\sqrt{2}}$ and so $\sin B = \frac{1}{2}$. We have two options, either $B = 30°$ or $B = 150°$.
   Are both possible? No, because $A + B + C = 180°$ and already $A + B = 195°$! So this means $B = 30°$. From here all the other sides and angles are easy.
   Notice that this is the one-solution case but we didn't need to know that going into the problem, we could figure it out.

   (b) Suppose a triangle has $C = 10°$, $c = 6$ and $b = 11$. Find B.
   By the LOS, $\frac{\sin 10°}{6} = \frac{\sin B}{11}$ and so $\sin B = \frac{11 \sin 10°}{6} \approx 0.3183549924$. We don't know this so we go to the calculator. $B = \sin^{-1}(0.3183549924) \approx 18.56°$. But this is not the only one, we could also have $B \approx 180 - 18.56 = 161.44°$.



   Are both possible? Sure because there are no angle summing problems.

   (c) Suppose a triangle has $B = 35°$, $b = 9$ and $a = 20$. Find A.
   By the LOS, $\frac{\sin 35°}{9} = \frac{\sin A}{20}$ and so $\sin A = \frac{20 \sin 35°}{9} \approx 1.274614303$. Before we leap to the calculator, note that this not possible because sine is always from -1 to 1. So there is no solution here.

In other words the general approach here is to solve for sin(whatever) first. If there are no solutions stop, otherwise use your brain to get the two solutions or use the $\sin^{-1}$ button if necessary. Determine if one or both of these solutions works.