1. **Introduction:** Now we’d like to finish up by looking at the SAS and SSS cases. Both of these cases use the law of cosines.

2. **Law of Cosines:** Suppose we have a triangle with sides \( a, b, c \) and angles \( A, B, C \), where the angles are opposite their corresponding sides.

   Then the Law of Cosines says all of the following:
   
   \[
   c^2 = a^2 + b^2 - 2ab \cos C
   \]
   \[
   b^2 = a^2 + c^2 - 2ac \cos B
   \]
   \[
   a^2 = b^2 + c^2 - 2bc \cos A
   \]

   Note that in truth these all essentially the same thing just with the variables switched around.

3. **SAS Case, Application:** Suppose you need to measure the distance from point \( A \) to point \( B \) as shown in order to run some wire over the house. To do so you use the additional point \( C \) and measure the distances and angle shown. Find the distance from \( A \) to \( B \).

   
   ![Diagram showing points A, B, and C, with distances and angles labeled.]

   4. **SSS Case:** The SSS case with the Law of Cosines is a bit like the SSA case for the Law of Sines. There may in this case be one or no solutions only though. We may also need the \( \cos^{-1} \) button on our calculator.

   Example: Suppose \( a = 5, b = 6 \) and \( c = 7 \). Find \( A \).

   The Law of Cosines states that \( c^2 = a^2 + b^2 - 2bc \cos A \) so then \( 49 = 25 + 36 - 2(5)(6) \cos A \) so then \( \cos A = \frac{1}{5} \). Using our calculator we find that \( A \approx 78.46^\circ \).

   Note that this is the only angle that works. Technically speaking the angle \( 360 - 76.48 \) does too but this is not allowed in a triangle. In general in this situation if an angle works then it is the only one.

   Example: Suppose \( a = 2, b = 3 \) and \( c = 7 \). Find \( B \).

   It’s worth noting here that common sense dictates that it’s not possible for one side of a triangle to be longer than the sum of the other two sides, but if we didn’t think of this then the Law of Cosines would tell us that \( 9 = 4 + 49 - 2(2)(7) \cos B \) and so \( \cos B = \frac{11}{7} \), which is impossible. There is no such triangle.
5. **A Combination:** If you have time, here is a problem based off an old final exam problem which uses practically everything we know about trigonometry.

Suppose you wish to measure the distance between two points $A$ and $B$ on an island but you can’t actually get to the island. Instead you mark out additional points $C$, $D$ and $E$.

![Diagram of points A, B, C, D, E with measurements](image)

Suppose you then measure all of the following. Note that all of these can be measured without going to the island.

(a) Angle $C = 70^\circ$.
(b) Angle $E = 60^\circ$.
(c) Angle $\angle ADB = 18^\circ$.
(d) Angle $\angle BDE = 55^\circ$.
(e) Distance from $C$ to $D$ is 8 yards.
(f) Distance from $D$ to $E$ is 20 yards.

There may be other ways to do this but my plan was:

(a) Find distance $AD$ using right triangle trig.
(b) Find distance $BD$ using the Law of Sines.
(c) Find distance $AB$ using the Law of Cosines.