MATH 115 Section 6.5 Lecture Notes

1. **Introduction:** Now we'd like to finish up by looking at the SAS and SSS cases. Both of these cases use the law of cosines.

2. **Law of Cosines:** Suppose we have a triangle with sides a, b, c and angles A, B, C, where the angles are opposite their corresponding sides.

   Then the Law of Cosines says all of the following:

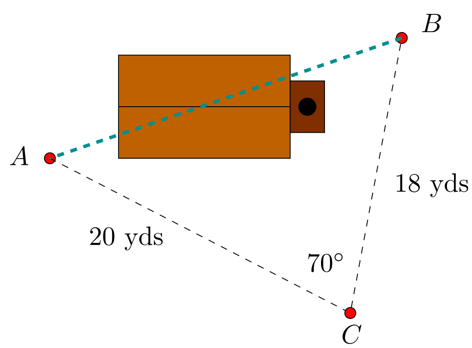
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

   Note that in truth these all essentially the same thing just with the variables switched around.

3. **SAS Case, Application:** Suppose you need to measure the distance from point A to point B as shown in order to run some wire over the house. To do so you use the additional point C and measure the distances and angle shown. Find the distance from A to B.



4. **SSS Case:** The SSS case with the Law of Cosines is a bit like the SSA case for the Law of Sines. There may in this case be one or no solutions only though. We may also need the $\cos^{-1}$ button on our calculator.

Example: Suppose $a = 5$, $b = 6$ and $c = 7$. Find A.

The Law of Cosines states that $c^2 = a^2 + b^2 - 2bc\cos A$ so then $49 = 25 + 36 - 2(5)(6)\cos A$ so then $\cos A = \frac{1}{5}$. Using our calculator we find that $A \approx 78.46°$.

Note that this is the only angle that works. Technically speaking the angle $360 - 76.48$ does too but this is not allowed in a triangle. In general in this situation if an angle works then it is the only one.

Example: Suppose $a = 2$, $b = 3$ and $c = 7$. Find B.

It's worth noting here that common sense dictates that it's not possible for one side of a triangle to be longer than the sum of the other two sides, but if we didn't think of this then the Law of Cosines would tell us that $9 = 4 + 49 - 2(2)(7)\cos B$ and so $\cos B = \frac{11}{7}$, which is impossible. There is no such triangle.

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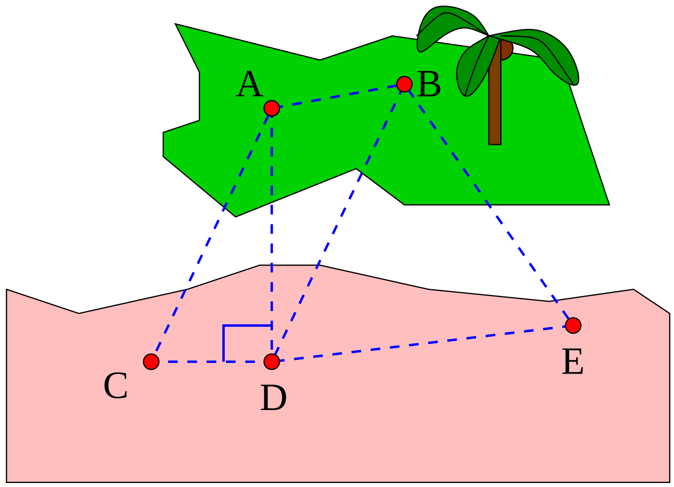
5. **A Combination:** If you have time, here is a problem based off an old final exam problem which uses practically everything we know about trigonometry.

   Suppose you wish to measure the distance between two points A and B on an island but you can't actually get to the island. Instead you mark out additional points C, D and E.



   Suppose you then measure all of the following. Note that all of these can be measured without going to the island.

   (a) Angle $C = 70°$.
   (b) Angle $E = 60°$.
   (c) Angle $\angle ADB = 18°$.
   (d) Angle $\angle BDE = 55°$.
   (e) Distance from C to D is 8 yards.
   (f) Distance from D to E is 20 yards.

   There may be other ways to do this but my plan was:

   (a) Find distance AD using right triangle trig.
   (b) Find distance BD using the Law of Sines.
   (c) Find distance AB using the Law of Cosines.