MATH 115 Section 7.1 Lecture Notes

---

1. **Introduction:** Now that we know how the trigonometric functions are defined, we'd like to look at some properties of them beyond the ones that we have already. The properties we'd like to look at are called *trigonometric identities* and we've seen a few of these already.

2. **Definition:** An *identity* is something which is always true, no matter what the value of the variable is.

   Example: The equation $2x + 3x = 5x$ is an identity. It is always true, no matter what value x takes.

   Example: The equation $2x+5 = 11$ is not an identity. It is only true when $x = 3$ and otherwise it is false.

   Example: The equation $\sin^2 x + \cos^2 x = 1$ is an identity. It is always true, no matter what value x takes.

   ---

3. **Trigonometric Identities:** We've seen several trigonometric identities, including all the reciprocal identities (like $\tan x = \frac{\sin x}{\cos x}$), the even-odd identities (like $\sin(-x) = -\sin x$) and the Pythagorean identities (like $1 + \tan^2 x = \sec^2 x$). It's also possible to prove some other ones. Typically this is done using the ones we already have.

   Easy Example: Show that $\tan x \csc x = \sec x$. An easy way to do this is to take the complicated side (the left side), convert it all to $\sin x$ and $\cos x$ using identities we know, then simplify it. When you're done you see that it's the same as the right side (by another identity).

   ---

4. **Proving Identities:** We will not do a lot of complicated proving of identities but we do need to learn how to simplify some commonly-appearing expressions. Here are a set of fundamental examples which will come up repeatedly in this course as well as in Calculus. In these examples we are creating an identity through the process of simplification.

   Example: Let $f(x) = \frac{x^2}{\sqrt{1-x^2}}$ and $g(u) = \sin u$. Suppose $-\frac{\pi}{2} < u < \frac{\pi}{2}$. Find $(f \circ g)(u)$ and simplify.

   Note: This does not even look like a trig identity but when you find $(f \circ g)(u)$ we get $\frac{\sin^2 u}{\sqrt{1-\sin^2 u}}$ which can be simplified. This creates a trig identity.

   Example: Simplify $\frac{\csc(-x)\cot(x)}{\sec(-x)}$ by converting everything to sines and cosines.

   Example: Convert $\tan x$ into an expression involving only $\cos x$. Assume that $\sin x < 0$.

   Example: Simplify $\frac{\sin x}{\sec x + \tan x}$, converting to only sines and cosines and eliminating the fraction.

   Example: Rewrite $\sin^2 x \cos^3 x$ in the form $A \cos x$, where A is an expression involving no trig functions except $\sin x$.
