MATH 115 Section 7.2 Lecture Notes

1. **Introduction:** Now that we know what an *identity* is, we'd like to look at some specific identities which are very useful and need to be memorized. In this section we will look at the addition and subtraction identities. We will prove only the first.

2. **Addition Formula for Coine:** The addition formula for sine states

$$\cos(a+b) = \cos a \cos b - \sin a \sin b.$$

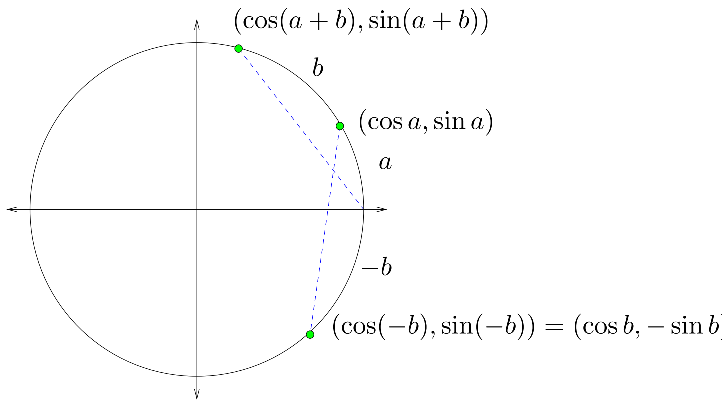
Before we explain why it's true, here is an example of why it's helpful. Suppose we wanted to know $\cos\left(\frac{5\pi}{12}\right)$. This is not one of our nice values. But observe that

$$\frac{5\pi}{12} = \frac{2\pi}{12} + \frac{3\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$$

(note that both of these values are nice) and so then by the formula

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4} = \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}.$$

3. **Proof:** Consider the following picture on which we've drawn the terminal points for $a+b$, b and $-b$.



Since the two measured distances are the same we have

$$\begin{aligned}
\sqrt{(\cos(a+b)-1)^2 + (\sin(a+b)-0)^2} &= \sqrt{(\cos a - \cos b)^2 + (\sin a + \sin b)^2} \\
(\cos(a+b)-1)^2 + (\sin(a+b)-0)^2 &= (\cos a - \cos b)^2 + (\sin a + \sin b)^2 \\
\cos^2(a+b) - 2\cos(a+b) + 1 + \sin^2(a+b) &= \cos^2 a - 2\cos a\cos b + \cos^2 b + \sin^2 a + 2\sin a \sin b + \sin^2 b \\
-2\cos(a+b) &= -2\cos a \cos b + 2 \sin a \sin b \\
\cos(a+b) &= \cos a \cos b - \sin a \sin b
\end{aligned}$$

as we wanted.

4. **A Summary of All:** A similar explanation can be found for the others but we will just state them. In the following I've used $\pm$ and $\mp$. Make sure you understand how they match.

   (a) $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

   (b) $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

   (c) $\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$

5. **A Number of Uses:** Here are lots of uses.

Example: Find $\sin\left(\frac{\pi}{12}\right)$. Note that this can be done two ways since $\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ or $\frac{\pi}{12} = \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$.

Example: Show that $\sin\left(x + \frac{\pi}{2}\right) = \cos x$.

Example: Rewrite $\cos\left(x + \frac{5\pi}{4}\right)$ in terms of only $\sin x$ and $\cos x$. Once this is done, assume that $\sin x = 0.2$ and $\cos x < 0$ and find a numerical value.

Example: Find the exact value of $\cos\left(\frac{9\pi}{11}\right)\cos\left(\frac{-16\pi}{33}\right) - \sin\left(\frac{9\pi}{11}\right)\sin\left(\frac{-16\pi}{33}\right)$ with no calculator.

Example: Find $\cot\left(\frac{11\pi}{12}\right)$.

Example: Find (but do not memorize) an addition formula for $\sec(a+b)$.
