MATH 115 Section 7.3 Lecture Notes

1. **Introduction:** More important identities! Note to the students and the TAs: We are not covering all of the identities in this section. They only need to know the double-angle identities and the power-reducing identities. It's also worth noting that because of the way we're covering the material, "double-angle" is sort of a misnomer because we haven't really formally dealt with trigonometric functions of angles yet.

2. **Double-Angle Identities:** These are some of the most important identities. They follow very simply from the addition identities but you should know them on their own. For this reason we will show how they are all derived:

   (a) For sine, observe that

   $$\begin{aligned}\sin(2x) &= \sin(x+x)\\ &= \sin x\cos x + \cos x\sin x\\ &= 2\sin x\cos x\end{aligned}$$

   (b) For cosine, observe that

   $$\begin{aligned}\cos(2x) &= \cos(x+x)\\ &= \cos x\cos x - \sin x\sin x\\ &= \cos^2 x - \sin^2 x\end{aligned}$$

   but we're not quite done. We can also get two more possibilities for cosine:

   $$\begin{aligned}&= \cos^2 x - (1-\cos^2 x)\\ &= 2\cos^2 x - 1\end{aligned}$$

   and

   $$\begin{aligned}&= (1-\sin^2 x) - \sin^2 x\\ &= 1 - 2\sin^2 x\end{aligned}$$

   (c) For tangent, observe that

   $$\begin{aligned}\tan(2x) &= \tan(x+x)\\ &= \frac{\tan x + \tan x}{1-\tan x\tan x}\\ &= \frac{2\tan x}{1-\tan^2 x}\end{aligned}$$

   In summary:

   (a) $\sin(2x) = 2\sin x\cos x$
   (b) $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
   (c) $\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$

3. **Examples:** Here are some basic examples:

Example: Suppose $\sin x = -0.2$ and the terminal point for x is in the fourth quadrant. Find $\sin(2x)$, $\cos(2x)$ and $\tan(2x)$.

Example: Suppose $\pi < x < 2\pi$. Write $\sin(2x)$ in terms of $\cos x$ only.

Example: Construct a double-angle formula for $\csc x$.

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4. **Power-Reducing Identities:** Observe that $1 - 2\sin^2 x = \cos(2x)$. If we solve for $\sin^2 x$ we get $\sin^2 x = \frac{1-\cos(2x)}{2}$. Alternately if we started with $2\cos^2 x - 1 = \cos(2x)$ we get $\cos^2 x = \frac{1+\cos(2x)}{2}$. These yield our first two power-reducing identities. The third, for $\tan^2 x$, comes from dividing the first two. In summary:

   (a) $\sin^2 x = \frac{1-\cos(2x)}{2}$

   (b) $\cos^2 x = \frac{1+\cos(2x)}{2}$

   (c) $\tan^2 x = \frac{1-\cos(2x)}{1+\cos(2x)}$

5. **Examples:** Some examples:

Example: Rewrite $\sin^4 x$ with no powers of trig functions.
