# Chapter 10 - Differentiation

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Differentiation is the most important thing in calculus so let's get to it!

## Using `diff`

What about calculus? Don't worry -- Matlab will not let you down! Suppose you'd like to differentiate the function $\log(6*x+2)$. You could either do it yourself or... just ask Matlab to do it with the `diff` command:

```matlab
syms x
diff(log(6*x+2))
ans =
6/(6*x + 2)
```

Or with a symbolic function:

```matlab
syms f(x);
f(x) = cos(x^2+2*tan(x));
diff(f(x))
ans =
-sin(2*tan(x) + x^2)*(2*x + 2*tan(x)^2 + 2)
```

It's okay if we don't plug the $x$ in when we write it:

```matlab
diff(f)
```

```matlab
ans(x) =
-sin(2*tan(x) + x^2)*(2*x + 2*tan(x)^2 + 2)
```

## Higher Derivatives

What could be easier?

Would you like to find the third derivative of the function $\log(6*x+2)$? That's easy too -- just pass 3 as a second parameter to the `diff` command:
diff(log(6 * x + 2), 3)

ans =
432/(6*x + 2)^3

**IMPORTANT**

It is a common mistake to believe that the 3 in the above calculation will take the first derivative and plug in x=3. It does not do this! If you want to differentiate and then plug in just wait a bit and we'll cover that.

**Higher Derivatives - An Alternate Way**

It's worth noting that we could have taken the third derivative this way, though we probably wouldn't:

```plaintext
diff(diff(diff(log(6*x+2)))
```

ans =
432/(6*x + 2)^3

**A Different Variable**

Suppose our expression has two variables and we want the derivative with respect to one of them. As usual x is the default

```plaintext
sym a x
diff(a^3*x^4)
```

ans =
4*a^3*x^3

but we can tell Matlab differently.

```plaintext
diff(a^3*x^4,a)
```

ans =
3*a^2*x^4

We can even do the second derivative with respect to a.

```plaintext
diff(a^3*x^4,a,2)
```

ans =
6*a*x^4

**Wait, that Second Parameter?**

Matlab is smart. If the second parameter is a variable it will take the derivative with respect to that variable. If it's a number it will take that numbered derivative. If it sees a variable and then a number it will take that numbered derivative with respect to that variable.

**Again with Symbolic Functions**

If we have a symbolic function of multiple variables we can differentiate too:
syms f(x,y);
f(x,y) = 2*x^2*y^3+x*sin(x*y);
diff(f(x,y),x)

ans =
sin(x*y) + 4*x*y^3 + x*y*cos(x*y)

We could even take the derivative with respect to $x$ and then with respect to $y$. This might only make sense to those with multidimensional calculus:

diff(diff(f(x,y),x),y)

ans =
2*x*cos(x*y) + 12*x*y^2 - x^2*y*sin(x*y)

Differentiating and then Plugging In - Using \texttt{subs}.

It may seem a bit late but this is the perfect time to talk about plugging things into symbolic expressions. Here's how. Suppose we simply want to plug $x=3$ into $x^2-x+2$. We do:

\texttt{subs(x^2-x+2,x,3)}

\texttt{ans} =
8

So now to take the derivative and then plug in, we simply nest the commands. Here's the second derivative of $x^3+\exp(x^2)$ with $x=1$ plugged in:

\texttt{subs(diff(x^3+\exp(x^2),2),x,1)}

\texttt{ans} =
6*\exp(1) + 6

Understand: $x^3+\exp(x^2)$ is treated as a symbolic expression by Matlab, it's not treated as a function. Then \texttt{diff(x^3+\exp(x^2),2)} is also treated as a symbolic expression, and \texttt{diff} does its job on that symbolic expression, giving back another symbolic expression, which goes into \texttt{subs}.

Or with a function:

\texttt{syms f(x);}  
\texttt{f(x) = 1/(x^2+3);}  
\texttt{subs(diff(f(x)),x,-3)}

\texttt{ans} =
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Understand: Even though we think of $f(x)$ as a function, really $f$ is a function and $f(x)$ is a symbolic expression, so \texttt{diff} works just fine, followed by \texttt{subs}.

Common Mistake

A common mistake is the following:
syms f(x)
f(x)=x^2+5*x+3;
diff(f(6))

ans =
0

Do you see the problem? It's the order in which things have been done. First 6 was plugged in, yielding a constant, then the derivative was taken, yielding 0. Instead as we've seen, we need to do:

subs(diff(f(x)),x,6)

ans =
17

Plotting Derivatives

Likewise we can nest diff inside fplot. Here's an example, a plot of the derivative of \( \sin(x^2) \):

fplot(diff(sin(x^2)))