241 Final Spring 2011 Notes

1. (a) Pick a point on the line and connect it to the given point to get one vector. Cross this with 
   the vector from the line itself to get the normal for the plane.
   
   (b) Formula!
   
   (c) Just use the gradient formula. Looks messy but isn’t, really.
   
   (d) Make sure your direction is a unit vector first, then dot it with the answer you got in (c).

2. Take the derivatives and set them equal to 0 to get the critical points. Plug each point into the 
   discriminant (and perhaps $f_{xx}$) to categorize.

3. Just use the formula for length of curve. The integral shouldn’t be bad if you’re careful.

4. Straightforward in spherical coordinates.

5. (a) FTOLI since the vector field is conservative.
   
   (b) Not a bad integral.

6. Parametrize $\Sigma$ with $\vec{r}(x,y)$ since you know $z$ in terms of $x$ and $y$. The $x,y$ are bounded by a 
   triangle. Then just go from there.

7. The surface $\Sigma$ is the part of the plane inside the cylinder and hence is best done in polar. The 
   curl is not bad and when you dot it with $\vec{r}_r \times \vec{r}_\theta$ you get a double integral that’s not so bad to 
   evaluate.

8. (a) This part is just calculation but it’s prepping you for (b).
   
   (b) The Divergence Theorem requires you to take the divergence of $\vec{F} \times \vec{G}$, but you got 0 in (a) 
   so this integral turns into an integral over $D$ of 0, hence is 0.