MATH 241 Final Examination

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Instructions. Answer each question **on a *separate* answer sheet**. Show all your work. A correct answer without work to justify it may not receive full credit. *Be sure your name, section number, and problem number are on each answer sheet, and that you have copied and signed the honor pledge on the first answer sheet.* The point value of each problem is indicated. The exam is worth a total of 200 points. In problems with multiple parts, whether the parts are related or not, be sure to go on to subsequent parts even if there is some part you cannot do. Please leave answers such as $5\sqrt{2}$ or 3π in terms of radicals and π and *do not convert to decimals.*
You are **not** allowed use of any notes. Calculators are **not** permitted.

1. (20 points, divided as indicated) Consider the points $P = (1, 0, -1)$, $Q = (-5, 3, 2)$, $R = (2, -1, 4)$. Suppose $\mathscr{P}$ is the plane that contains all these three points, and ℓ is the line through P perpendicular to the plane $\mathscr{P}$.
   (a) (5 points) Determine the equation of the plane $\mathscr{P}$.
   (b) (5 points) Find the parametric equations of the line ℓ.
   (c) (10 points) Find the equation of the sphere centered at the origin that is tangent to the plane $\mathscr{P}$.

2. (25 points, divided as indicated) An object moves along the curve C parametrized by
$$\mathbf{r}(t) = e^{2t}\,\mathbf{i} + 2\sqrt{2}e^{t}\,\mathbf{j} + 2t\,\mathbf{k} \qquad \text{for } t \geq 0.$$
   (a) (4 points) Find the velocity and acceleration of this object as functions of time.
   (b) (11 points) What is the distance traveled by the object along the curve C from $t = 0$ to $t = 1$?
   (c) (10 points) Determine the tangential and normal components of the acceleration, a_T and a_N.

3. (20 points, divided as indicated) The function $u(t, x)$ satisfies the wave equation if $c^{-2}u_{tt} - u_{xx} = 0$ for all (t, x), where c is a constant.
   (a) (5 points) Does $u(t, x) = 10\cos(3x - 3ct)$ satisfy the wave equation?
   (b) (5 points) Does $u(t, x) = 10\cos(3x - 2ct)$ satisfy the wave equation?
   (c) (5 points) Does $u(t, x) = e^{x-ct}$ satisfy the wave equation?
   (d) (5 points) Does $u(t, x) = (x + ct)^5$ satisfy the wave equation?

4. (25 points) Find the critical points of the function $f(x, y) = x^4 - 2x^2 - 3xy^2 + 9$. By applying the second partials test, classify each critical point as a local (relative) maximum point, local (relative) minimum point, saddle point, or something else. (If this test is inconclusive, simply write so.)

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5. (25 points) By using a suitable change of variables, evaluate the double integral

$$\iint_R \left(1+\frac{x^2}{4}+\frac{y^2}{25}\right)^{1/2} dA,$$

where R is the region bounded by the ellipse $x^2/4 + y^2/25 = 1$.

6. (30 points, divided as indicated)

   (a) (10 points) Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = x\,\mathbf{i} + z\,\mathbf{j} + x^2 z\,\mathbf{k}$ and C is the curve parametrized by

   $$\mathbf{r}(t) = t\,\mathbf{i} + \cos t\,\mathbf{j} + \mathbf{k}, \qquad 0 \le t \le \pi.$$

   (b) (20 points) Use Green's theorem to compute the line integral

   $$\int_C (xy + \sqrt{1+x^2})\,dx + (xy + \cos y)\,dy,$$

   where C is the circle $(x-2)^2 + y^2 = 4$ oriented counter-clockwise.

7. (25 points) Evaluate the surface integral $\iint_\Sigma \mathbf{F}\cdot\mathbf{n}\,dS$, where

$$\mathbf{F}(x,y,z) = e^y\,\mathbf{i} + (y+2x)\,\mathbf{j} + z\,\mathbf{k},$$

Σ is the portion of the plane $2x + y + 2z = 4$ in the first octant, and $\mathbf{n}$ is the normal unit vector pointing upward.

8. (30 points) Use spherical coordinates to compute the total mass M of an object occupying the solid region D that is bounded above by the sphere $x^2 + y^2 + z^2 = 4z$ and below by the cone $z = \sqrt{x^2+y^2}/\sqrt{3}$. Assume that the mass density of the object at the point (x,y,z) of D is equal to the distance from (x,y,z) to the origin.