Math 241 Spring 2013 Final Exam

- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets.
- No calculators are permitted.
- One page of notes is permitted.
- Do not evaluate integrals or simplify answers unless indicated.

Please put problem 1 on answer sheet 1

1. (a) Find the distance from the point $(1, -2, 2)$ to the plane $3x + y - 4z = 2$. [5 pts]
   (b) Find all possible $x$ so that $3i + 5j$ is parallel to $5z^2i + 10j$. [5 pts]
   (c) Find the tangential component of acceleration for $r(t) = 2t^2i + \frac{1}{t}j + t\ k$ at $t = -1$. [10 pts]

Please put problem 2 on answer sheet 2

2. (a) Find the equation of the plane containing both the point $(0, 1, -2)$ and the line with symmetric equations

\[
\frac{x - 1}{2} = y + 2, \quad z = 3
\]

(b) Find the tangent vector $T(\pi/6)$ for the curve $r(t) = 2\cos(t)i + 3\sin(2t)j$. [10 pts]

Please put problem 3 on answer sheet 3

3. Define $f(x, y) = x^2y + 3y$.
   (a) Find the vector equation of the line perpendicular to the level curve of $f$ at $(2, -1)$. [10 pts]
   (b) Find the value of the maximal directional derivative of $f$ at $(2, -1)$. [4 pts]
   (c) If $u$ is a unit vector which makes an angle of $\pi/3$ with $\nabla f$ at $(2, -1)$, find $D_uf(2, -1)$. [6 pts]

Please put problem 4 on answer sheet 4

4. Let $f(x, y) = 2x^3 - 24x + 2y^3 - 3y^2 - 12y - 1$. Find all critical points for $f$ and determine whether each critical point yields a relative maximum, relative minimum or saddle point. [20 pts]

Turn Over!
Please put problem 5 on answer sheet 5

5. Use the method of Lagrange multipliers to find the maximum and minimum values of the function \( f(x, y) = xy + 2x \) on the circle \( x^2 + y^2 = 4 \). [20 pts]

Please put problem 6 on answer sheet 6

6. (a) Evaluate \( \int_0^1 \int_x^1 \cos(y^2) \, dy \, dx \). [10 pts]

(b) Set up the iterated integral in polar coordinates for \( \iint_R xy \, dA \) where \( R \) is the region inside the circle \( r = 2 \sin \theta \) and above the line \( y = 1 \). Do not evaluate. [10 pts]

Please put problem 7 on answer sheet 7

7. Evaluate \( \int_C x^2 \, dx + 5xy \, dy \) where \( C \) is the triangle with vertices \((0, 0), (6, 3) \) and \((6, 6) \) with counterclockwise orientation. [20 pts]

Please put problem 8 on answer sheet 8

8. (a) Evaluate the line integral \( \int_C x \, ds \) where \( C \) is the straight line segment from \((1, 2)\) to \((5, 10)\). [10 pts]

(b) Evaluate the line integral \( \int_C F \cdot dr \) where \( C \) is the curve \( r(t) = (t^2 + t) \, i + (\frac{1}{2}) \, j + (t^3 - 2t + 1) \, k \) for \( 1 \leq t \leq 2 \) and \( F(x, y, z) = (2xy + z) \, i + (x^2 + 3) \, j + z \, k \). [10 pts]

Please put problem 9 on answer sheet 9

9. Let \( C \) be the intersection curve of the cylinder \( x^2 + y^2 = 9 \) with the plane \( z = 10 - y \) and with clockwise orientation when viewed from above. Use Stokes' Theorem to convert \( \int_C (xz \, i + z^2 \, j + y \, k) \cdot dr \) to a surface integral. Then proceed until you have an iterated integral but do not evaluate. [20 pts]

Please put problem 10 on answer sheet 10

10. Evaluate \( \iint_\Sigma (2 \, i + 4 \, j + z^2 \, k) \cdot \mathbf{n} \, dS \) where \( \Sigma \) is the part of the cone \( z = \sqrt{3x^2 + 3y^2} \) inside the sphere \( x^2 + y^2 + z^2 = 9 \) as well as the part of the sphere inside the cone, with inward orientation. [20 pts]

Welcome to the End of the Exam