Math 241 Spring 2013 Final Exam

- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets.
- No calculators are permitted.
- One page of notes is permitted.
- Do not evaluate integrals or simplify answers unless indicated.

Please put problem 1 on answer sheet 1

1. (a) Find the distance from the point $(1,-2,2)$ to the plane $3x+y-4z=2$. [5 pts]

   (b) Find all possible x so that $3\,\mathbf{i}+5\,\mathbf{j}$ is parallel to $5x^2\,\mathbf{i}+10\,\mathbf{j}$. [5 pts]

   (c) Find the tangential component of acceleration for $\mathbf{r}(t)=2t^2\,\mathbf{i}+\frac{1}{t}\,\mathbf{j}+t\,\mathbf{k}$ at $t=-1$. [10 pts]

Please put problem 2 on answer sheet 2

2. (a) Find the equation of the plane containing both the point $(0,1,-2)$ and the line with symmetric equations [10 pts]

$$\frac{x-1}{2}=y+2\ ,\quad z=3$$

   (b) Find the tangent vector $\mathbf{T}(\pi/6)$ for the curve $\mathbf{r}(t)=2\cos(t)\,\mathbf{i}+3\sin(2t)\,\mathbf{j}$. [10 pts]

Please put problem 3 on answer sheet 3

3. Define $f(x,y)=x^2y+3y$.

   (a) Find the vector equation of the line perpendicular to the level curve of f at $(2,-1)$. [10 pts]

   (b) Find the value of the maximal directional derivative of f at $(2,-1)$. [4 pts]

   (c) If $\mathbf{u}$ is a unit vector which makes an angle of $\pi/3$ with ∇f at $(2,-1)$, find $D_{\mathbf{u}}f(2,-1)$. [6 pts]

Please put problem 4 on answer sheet 4

4. Let $f(x,y)=2x^3-24x+2y^3-3y^2-12y-1$. Find all critical points for f and determine whether each critical point yields a relative maximum, relative minimum or saddle point. [20 pts]

Turn Over!

Please put problem 5 on answer sheet 5

5. Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x,y) = xy + 2x$ on the circle $x^2 + y^2 = 4$. [20 pts]

Please put problem 6 on answer sheet 6

6. (a) Evaluate $\int_0^1 \int_x^1 \cos(y^2)\ dy\ dx$. [10 pts]

   (b) Set up the iterated integral in polar coordinates for $\iint\limits_R xy\ dA$ where R is the region inside the circle $r = 2\sin\theta$ and above the line $y = 1$. Do not evaluate. [10 pts]

Please put problem 7 on answer sheet 7

7. Evaluate $\int\limits_C x^2\ dx + 5xy\ dy$ where C is the triangle with vertices $(0,0)$, $(6,3)$ and $(6,6)$ with counterclockwise orientation. [20 pts]

Please put problem 8 on answer sheet 8

8. (a) Evaluate the line integral $\int\limits_C x\ ds$ where C is the straight line segment from $(1,2)$ to $(5,10)$. [10 pts]

   (b) Evaluate the line integral $\int\limits_C \mathbf{F}\cdot d\mathbf{r}$ where C is the curve $\mathbf{r}(t) = (t^2+t)\,\mathbf{i}+(\frac{1}{t})\,\mathbf{j}+(t^3-2t+1)\,\mathbf{k}$ for $1 \le t \le 2$ and $\mathbf{F}(x,y,z) = (2xy+z)\,\mathbf{i} + (x^2+3)\,\mathbf{j} + x\,\mathbf{k}$. [10 pts]

Please put problem 9 on answer sheet 9

9. Let C be the intersection curve of the cylinder $x^2 + y^2 = 9$ with the plane $z = 10 - y$ and with clockwise orientation when viewed from above. Use Stokes' Theorem to convert $\int\limits_C (xz\,\mathbf{i} + z^2\,\mathbf{j} + y\,\mathbf{k})\cdot d\mathbf{r}$ to a surface integral. Then proceed until you have an iterated integral but do not evaluate. [20 pts]

Please put problem 10 on answer sheet 10

10. Evaluate $\iint\limits_\Sigma (2\,\mathbf{i} + 4\,\mathbf{j} + z^2\,\mathbf{k})\cdot\mathbf{n}\ dS$ where Σ is the part of the cone $z = \sqrt{3x^2 + 3y^2}$ inside the sphere $x^2 + y^2 + z^2 = 9$ as well as the part of the sphere inside the cone, with inward orientation. [20 pts]

Welcome to the End of the Exam