241 Final Spring 2013 Notes

1. (a) Know the formula! You'll need a point on the plane, any point satisifying $3x + y - 4z = 2$ works. Pick something simple.
   (b) Parallel means they're multiples of one another. Because of the $5\,\hat{\jmath}$ and $10\,\hat{\jmath}$ the second must be twice the first.
   (c) Know the formula!
2. (a) For the plane you'll need a point and a normal vector. You have a point given. For the normal vector you'll need to take a cross product of two vectors parallel to the plane. Use the direction vector for the line for one of them and for another vector take a point on the line (any) and connect it to $(0, 1, -2)$.
   (b) Know the formula!
3. (a) Take the gradient of f to get a vector perpendicular to the level curve and then use this vector and $(2, -1)$ to construct the line.
   (b) This is just the magnitude of the gradient.
   (c) The directional derivative is the dot product of $\bar{u}$ with the gradient and the dot product can be written with a cosine.
4. Take the derivatives and set them equal to 0 to get the critical points. Plug each point into the discriminant (and perhaps f_{xx}) to categorize.
5. The constraint function is $g(x, y) = x^2 + y^2$. Write down the three equations $f_x = \lambda g_x$, $f_y = \lambda g_y$ and $x^2 + y^2 = 4$ and solve for all possible (x, y). Plug these into f and select the largest and smallest.
6. (a) You'll need to change the order of integration.
   (b) Draw a picture, it'll help, and don't forget to change $y = 1$ to polar for your outside function.
7. Green's Theorem, it's pretty direct.
8. (a) There's only one way to do this. Parametrize the line segment $\bar{r}(t) = (1 + 4t)\,\hat{\imath} + (2 + 8t)\,\hat{\jmath}$ and go from there using the only way.
   (b) The complicated nature of this parametrization is a hint. The vector field is conservative so use FTOLI.
9. Convert by Stokes' to a surface integral where Σ is the portion of the plane inside the cylinder. Parametrize this Σ using $\bar{r}(\theta, r)$.
10. This is the Divergence Theorem since Σ surrounds the solid D inside the cone and inside the sphere. Set this up with spherical coordinates, the integral is not bad at all.