241 Final Spring 2014 Notes

1. (a) Find the vector (subtract) and use either point.
   (b) Know the formula. You'll need a point on the plane, any point satisfying the plane equation will work.

2. (a) Formula!
   (b) Formula! Know the nice one with $\bar{v}$ and $\bar{a}$.

3. (a) Formula!
   (b) Formula!
   (c) Plug the components of $\bar{r}$ into the plane equation and solve to get t. Put this back into $\bar{r}$ to get the point.

4. The constraint function is $g(x, y) = (x - 2)^2 + y^2$. Write down the three equations $f_x = \lambda g_x$, $f_y = \lambda g_y$ and $(x - 2)^2 + y^2 = 4$ and solve for all possible (x, y). Plug these into f and select the largest and smallest.

5. Take the derivatives and set them equal to 0 to get the critical points. Plug each point into the discriminant (and perhaps f_{xx}) to categorize.

6. (a) Take the gradient and divide by its magnitude.
   (b) This is just the magnitude from the previous part.
   (c) Note the point changed. Take the gradient at this new point and dot it with the direction but make the direction a unit vector first.

7. (a) Best is $\bar{r}(x, \theta)$.
   (b) First write the cylinder as the level surface for $f(x, y, z) = y^2 + z^2$. Take the gradient to get the normal vector and then use the point to build the plane.

8. Use spherical coordinates. This cone is $\phi = \pi/3$.

9. The surface Σ is the part of the parabolic sheet inside the cylinder. Parametrize this with $\bar{r}(x, z)$ because you know $y = x^2$.

10. (a) Green's Theorem for sure, and the $N_x - M_y$ is nice!
    (b) Surface area is the surface integral $\iint_\Sigma 1\ dS$ and the only way to do this is to parametrize Σ. Complete the square on the cylinder and then Σ is nice in polar.