Math 241 Spring 2015 Final Exam

- Be sure your name, section number and problem number are on each answer sheet and that you have copied and signed the honor pledge on the first answer sheet.
- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets but if you do so, please write "See Back" or something similar on the bottom of the front so we know!
- No calculators or formula sheets are permitted.
- For problems with multiple parts, whether the parts are related or not, be sure to go on to subsequent parts even if there is some part you cannot do.
- Please leave answers such as $5\sqrt{2}$ or 3π in terms of radicals and π and *do not convert to decimals.*
- Numerical answers do not need to be simplified.

Please put problem 1 on answer sheet 1

1. Find the equation of the plane containing the line $x = y - 1 = \frac{z+1}{2}$ and the point $(1, 0, 1)$. Write your final answer in the form $ax + by + cz = d$. [20 pts]

Please put problem 2 on answer sheet 2

2. Consider the planes given by the equations $x + 2y - z = 1$ and $2x + y + z = -1$.
   (a) Find a point on the intersection of the planes. [8 pts]
   (b) Find the symmetric form of the equation of the line formed by the intersection of the planes. [12 pts]

Please put problem 3 on answer sheet 3

3. Let $\mathbf{r}(t) = \sin t\,\mathbf{i} + \cos t\,\mathbf{j} + \frac{2}{3}t^{3/2}\,\mathbf{k}$ for $0 \le t \le 1$ be the parametrization of a curve.
   (a) Determine whether the curve is smooth, piecewise smooth or neither. [8 pts]
   (b) Find the length of the curve. [12 pts]

Please put problem 4 on answer sheet 4

4. Find the tangent and normal vectors for the curve [20 pts]

$$\mathbf{r}(t) = \cos(2t)\,\mathbf{i} + \cos(2t)\,\mathbf{j} + \sqrt{2}\sin(2t)\,\mathbf{k}$$

Please put problem 5 on answer sheet 5

5. Find all points on the graph of $z = xy(1 - x - y)$ where the tangent plane is parallel to the xy-plane. [20 pts]

Turn Over!

Please put problem 6 on answer sheet 6

6. Use Lagrange Multipliers to find the maxima and minima of the function $f(x, y) = x^2 + xy + y^2$ subject to the constraint $x^2 + y^2 = 8$. [20 pts]

Please put problem 7 on answer sheet 7

7. Find the volume V of the solid region D bounded above by the plane $z = 4 + x + 2y$, on the sides by the cylinder $x^2 + y^2 = 4$, and below by the xy-plane. [20 pts]

Please put problem 8 on answer sheet 8

8. By using a suitable change of variables, evaluate [20 pts]

$$I = \iint_R \left(\frac{x - 3y}{x + 3y} \right)^2 \, dA$$

where R is the region bounded by the lines $x - 3y = 1$, $x - 3y = 2$, $x + 3y = 1$, $x + 3y = 3$.

Please put problem 9 on answer sheet 9

9. Let C be the intersection curve of the cylinder $(x-1)^2 + y^2 = 1$ and the paraboloid $z = 9 - x^2 - y^2$ with counterclockwise orientation when viewed from above. Use Stokes' Theorem to convert the line integral [20 pts]

$$\int_C xy \; dx + y \; dy + xz \; dz$$

to a surface integral. Parametrize and proceed until you have an iterated double integral. Do not evaluate this integral.

Please put problem 10 on answer sheet 10

10. (a) Let C be the curve parametrized by $\mathbf{r}(t) = t\,\mathbf{i} + 3t^2\,\mathbf{j} + (2t + 1)\,\mathbf{k}$ for $1 \leq t \leq 3$ and let the vector field $\mathbf{F}$ be defined by $\mathbf{F}(x, y, z) = 2x\,\mathbf{i} + \frac{1}{z}\,\mathbf{j} - \frac{y}{z^2}\,\mathbf{k}$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$. [10 pts]

    (b) Calculate $\int_C y \; dx + (x^2 + x) \; dy$ where C is the square with corners $(0, 0)$, $(4, 0)$, $(4, 2)$ and $(0, 2)$, oriented clockwise. [10 pts]

Welcome to the End of the Exam