MATH241 FINAL EXAM, Fall 2016

The test has 10 questions for a total of 200 points.

Instructions :

- Number the answer sheets from 1 to 10. Write your **name, section number and your TAs name** on each answer sheet (write and sign the Honor Pledge on page 1 only).
- **Answer question # 1 on answer sheet # 1, question # 2 on answer sheet # 2, etc.**
- This is a closed book exam, calculators and electronic devices are not permitted.
- You may continue your answers on the back of answer sheets, but please be sure to indicate that there is work on the back.
- Show all your work in order to receive full credit. Answers that are unjustified will receive no credit.
- Simplify all answers, unless otherwise indicated.

---

(1) a) (10 pts) Find an equation of the plane that contains the two parallel lines

line L_1 described by : $\dfrac{x-1}{3}=\dfrac{y+1}{2}=\dfrac{z-5}{4}$ and line L_2 described by : $\dfrac{x+3}{3}=\dfrac{y-4}{2}=\dfrac{z}{4}$.

b) (10 pts) Find the symmetric (or nonparametric) equation of a line that is orthogonal to the plane that you found in part (a) and passes through the point $(1,-1,5)$.

---

(2) Let L be the line described by $(x+1)/2 = (y+3)/3 = -z$ and let $\mathcal{P}$ be the plane described by $3x-2y+4z=-1$.
a) (10 pts) Find the point of intersection, call it P_0, of the line L with the plane $\mathcal{P}$.
b) (10 pts) Find an equation of the plane perpendicular (orthogonal) to line L at the point $(-1,-3,0)$.

---

(3) The velocity vector of a particle in space is given by $\vec{v}(t) := t\vec{i} + \sqrt{2}\vec{j} + (1/t)\vec{k}$.
a) (10 pts) What is the length of the curve traveled by this particle from $t=2$ to $t=4$?
b) (10 pts) If the position of this particle at $t=1$ is given by $\vec{r}(1) = 1\vec{i} + 2\vec{k}$, find the position at $t=2$ i.e. compute $\vec{r}(2)=?$.

---

(4) Let the function f be defined by $f(x,y,z) := x^2y + z^2$.
a) (10 pts) Find the directional derivative of the function f at the point $(1,2,3)$ in the direction of the unit vector $\vec{u} := \vec{a}/\|\vec{a}\|$ where the vector $\vec{a}$ is $\vec{a} := \vec{i} - \vec{j} + \vec{k}$.
b) (10 pts) Find the equation of the tangent plane to the level surface defined by $f(x,y,z)=5$, at the point with coordinates $(2,1,-1)$.

---

(5) Consider the following function of two variables $f(x,y) := \sin(x+y) + \cos(x-y)$ defined in the rectangle $D := \{(x,y) \mid 0 < x < \pi/2\ ,\ 0 < y < \pi/2\}$.
a) (10 pts) Find all critical points of this function inside D.
b) (10 pts) Determine if the critical points that you found in part (a) are local maxima, local minima, saddle points or undecided.

---

(6) Suppose that you want to find extrema (maxima or minima) of the function $f(x,y,z) := 4x + 6y + 10z$ under the restriction, $x^2+y^2+z^2 = 38$. You may assume that extrema exist.
a) (10 pts) Set up the Lagrange multiplier problem that you need to solve in order to determine the extrema.
b) (10 pts) Solve the resulting system from part (a) and find the maximum and the minimum.

TURN THE PAGE

(7) (20 pts) Compute the double integral below,

$$\int\int_D \cos(x+y)dA\ ,$$

where D is the triangle with vertices $(0,0)$, $(\pi/2, 0)$ and $(\pi/2, \pi/2)$.

---

(8) You want to compute the triple integral below,

$$\int\int\int_D \frac{z}{\sqrt{x^2+y^2}}dV$$

where D is the region described by $D := \{(x,y,z) \mid x^2+y^2+z^2 \leq 9\ ,\ z \geq \sqrt{x^2+y^2}\}$.

a) (10 pts) Use spherical coordinates in order to express the integral as an iterated integral in these coordinates. Indicate the limits of integration. Hint : The spherical coordinates are $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$ and $z = \rho\cos(\phi)$.

b) (10 pts) Compute the integral.

---

(9) (20 pts) Use Green's theorem in order to calculate the integral,

$$\int_C (e^x - 4y\sin^2(x))dx + (2x + 2\sin(x)\cos(x))dy$$

where C is the triangle with vertices $(1,1)$, $(1,2)$ and $(2,3)$ oriented counterclockwise.

---

(10) (20 pts) Use the divergence theorem in order to calculate the integral $\int\int_\Sigma (\vec{F}\cdot\vec{n})dS$, where Σ is the boundary of the upper half hemisphere of radius 2 i.e. Σ is the boundary of the region D described by $D := \{(x,y,z) \mid x^2+y^2+z^2 \leq 4\ ,\ z \geq 0\}$ and $\vec{F}$ is the vector-field $\vec{F} = x^2\vec{i} + 2yz\vec{j} - z^2\vec{k}$, with $\vec{n}$ directed outwards from the region D. Hint: You may use either spherical or cylindrical coordinates in order to compute the resulting triple integral. The cylindrical coordinates are $x = r\cos(\theta)$, $y = r\sin(\theta)$ and $z = z$.

THE END