Math 241 Fall 2016 Final Exam Solutions Partial

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4. Let $f(x,y,z) = x^2y + z^2$.

   (a) Find the directional derivative at $(1,2,3)$ in the direction of the unit vector $\mathbf{u} = \mathbf{a}/||\mathbf{a}||$ where $\mathbf{a} = 1\,\mathbf{i} - 1\,\mathbf{j} + 1\,\mathbf{k}$.
   Solution: We have:

   $$\mathbf{u} = \frac{1}{\sqrt{3}}\,\mathbf{i} - \frac{1}{\sqrt{3}}\,\mathbf{j} + \frac{1}{\sqrt{3}}\,\mathbf{k}$$

   and we have:

   $$\begin{aligned} f_x &= 2xy \\ f_y &= x^2 \\ f_z &= 2z \end{aligned}$$

   and so:

   $$\begin{aligned} D_{\mathbf{u}}f &= \frac{1}{\sqrt{3}}(2xy) - \frac{1}{\sqrt{3}}(x^2) + \frac{1}{\sqrt{3}}(2z) \\ D_{\mathbf{u}}f(1,2,3) &= \frac{1}{\sqrt{3}}(4) - \frac{1}{\sqrt{3}}(1) + \frac{1}{\sqrt{3}}(6) \end{aligned}$$

   (b) Find the equation of the tangent plane to the level surface defined by $f(x,y,z) = 5$ at the point $(2,1,-1)$.
   Solution: We have:

   $$\begin{aligned} \nabla f &= 2xy\,\mathbf{i} + x^2\,\mathbf{j} + 2z\,\mathbf{k} \\ \nabla f(2,1,-1) &= 4\,\mathbf{i} + 4\,\mathbf{j} - 2\,\mathbf{k} \end{aligned}$$

   So the plane is:

   $$4(x-2) + 4(y-1) - 2(z+1) = 0$$