Math 241 Fall 2017 Final Exam

- Be sure your name, section number and problem number are on each answer sheet and that you have copied and signed the honor pledge on the first answer sheet.
- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets but if you do so, please write "See Back" or something similar on the bottom of the front so we know!
- No calculators or formula sheets are permitted.
- For problems with multiple parts, whether the parts are related or not, be sure to go on to subsequent parts even if there is some part you cannot do.
- Please leave answers such as $5\sqrt{2}$ or 3π in terms of radicals and π and *do not convert to decimals.*

Please put problem 1 on answer sheet 1

1. (a) Find the cosine of the angle between the vectors $\mathbf{a} = 2\,\mathbf{i} - 1\,\mathbf{j} + 3\,\mathbf{k}$ and $\mathbf{b} = 0\,\mathbf{i} + 4\,\mathbf{j} + 2\,\mathbf{k}$. [5 pts]
Simplify your answer as much as possible.

   (b) Find the distance from the point $(3, 1, -1)$ to the plane with equation $2x - y - 2z = 4$. [20 pts]
Simplify your answer as much as possible.

Please put problem 2 on answer sheet 2

2. Find the equation of the line of intersection between the planes $x + 4z = 1$ and $-x + 2y + 3z = 5$. [20 pts]

Please put problem 3 on answer sheet 3

3. The position of a particle at time $t \geq 0$ is given by $\mathbf{r}(t) = 2t\,\mathbf{i} + t^2\,\mathbf{j} - \frac{1}{3}t^3\,\mathbf{k}$.

   (a) Find the length of the curve traced out by the particle in the time interval $0 \leq t \leq 3$. [10 pts]
Simplify your answer as much as possible.

   (b) Find the the tangential component of acceleration. [5 pts]
Simplify your answer as much as possible.

   (c) Is the speed of the particle increasing or decreasing when $t = 3$? Explain. [5 pts]

Please put problem 4 on answer sheet 4

4. Parts (a) and (b) are independent problems.

   (a) Let $w = f(x, y)$ where $x = e^{-s}\cos t$ and $y = e^{-s}\sin t$, and $f(x, y)$ is a function with continuous partial derivatives of first and second order. Compute both the partial derivatives w_s and w_t in terms of x, y, f_x and f_y. [10 pts]

   (b) Find a unit vector perpendicular to the graph of $f(x, y) = xy + xy^2$ at the point where $x = 1$ and $y = -2$. [10 pts]
Simplify your answer as much as possible.

Turn Over!

Please put problem 5 on answer sheet 5

5. Consider the function $f(x,y) = 3x^2 + 2y^2 - 8y + 1$. Find the maximum and minimum values of f on the region R of the xy-plane described by $x^2 + y^2 \le 1$. [20 pts]

Please put problem 6 on answer sheet 6

6. Parts (a) and (b) are independent problems.

   (a) Set up a triple iterated integral that would compute the volume of the solid region under the paraboloid $z = 9 - x^2 - y^2$, above the plane $z = 1$, and for $y \ge 0$. [10 pts]
   Do not evaluate this integral.

   (b) Set up a triple iterated integral that would determine the volume of the solid region below $z = x^4 + y^4 + 4$ and above the triangular region on the xy-plane with vertices $(-2,0)$, $(2,0)$, and $(0,2)$. [10 pts]
   Do not evaluate this integral.

Please put problem 7 on answer sheet 7

7. Use an appropriate change of variables to evaluate [25 pts]

$$\iint_R (3x + 6y)^2 \, dA$$

where R is the parallelogram enclosed by the lines $x - 2y = -2$, $x - 2y = 2$, $\frac{1}{2}x + y = 1$, and $\frac{1}{2}x + y = -1$.
Evaluate and simplify your answer as much as possible.

Please put problem 8 on answer sheet 8

8. Parts (a) and (b) are independent problems.

   (a) Let C be the clockwise curve consisting of the quarter-circle $x^2 + y^2 = 4$ with $x, y \ge 0$ along with the line segment joining $(0,0)$ to $(0,2)$ and the line segment joining $(0,0)$ to $(2,0)$. Use Green's Theorem to evaluate $\int_C 2x\,dx + x^2\,dy$. [15 pts]
   Evaluate and simplify your answer as much as possible.

   (b) Let Σ be the part of the plane $2x + y + 3z = 12$ in the first octant. If the mass density at any point is given by $f(x,y,z) = xz$, set up a double iterated integral for the mass of Σ. [10 pts]
   Do not evaluate this integral.

Please put problem 9 on answer sheet 9

9. Parts (a) and (b) are independent problems.

   (a) Let C be the intersection of the cylinder $x^2 + z^2 = 4$ with the plane $y + z = 8$ and with counterclockwise orientation when viewed from above. Use Stokes' Theorem to convert the line integral [20 pts]

$$\int_C x^2 z\,dx + x\,dy + yz\,dz$$

   to a surface integral. Parametrize the surface to obtain a double iterated integral.
   Do not evaluate this integral.

   (b) Evaluate $\iint_\Sigma (2x\,\mathbf{i} + 3x\,\mathbf{j} - 5z\,\mathbf{k}) \cdot \mathbf{n}\,dS$ where Σ is the sphere $x^2 + y^2 + z^2 = 9$ with $\mathbf{n}$ the unit normal vector pointing inwards. That is, the sphere is oriented inwards. [5 pts]
   Evaluate and simplify your answer as much as possible.

Welcome to the End of the Exam