Math 241 Spring 2017 Final Exam

- Be sure your name, section number and problem number are on each answer sheet and that you have copied and signed the honor pledge on the first answer sheet.
- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets but if you do so, please write "See Back" or something similar on the bottom of the front so we know!
- No calculators or formula sheets are permitted.
- For problems with multiple parts, whether the parts are related or not, be sure to go on to subsequent parts even if there is some part you cannot do.
- Please leave answers such as $5\sqrt{2}$ or 3π in terms of radicals and π and *do not convert to decimals.*
- Numerical answers do not need to be simplified.

Please put problem 1 on answer sheet 1

1. (a) Find $\cos\theta$ where θ is the angle between the vectors $\mathbf{a} = 2\,\mathbf{i} + 1\,\mathbf{j} - 3\,\mathbf{k}$ and $\mathbf{b} = 1\,\mathbf{i} + 5\,\mathbf{j} - 4\,\mathbf{k}$. [8 pts]

   (b) Find the distance between the point $(5, 12, -13)$ and the plane containing the points $(1, 1, 1)$, $(7, -1, -1)$, and $(0, 3, 0)$. [12 pts]

Please put problem 2 on answer sheet 2

2. Find the length of the curve $\mathbf{r}(t) = \frac{1}{2}t^2\,\mathbf{i} + \ln(t)\,\mathbf{j} + t\sqrt{2}\,\mathbf{k}$ from the point $\left(\frac{1}{2}, 0, \sqrt{2}\right)$ to the point $\left(2, \ln(2), 2\sqrt{2}\right)$. [20 pts]

Please put problem 3 on answer sheet 3

3. A particle with initial position $(0, 0, 1)$ has velocity $\mathbf{v}(t) = \frac{3}{2}(t+1)^{1/2}\,\mathbf{i} + e^{-t}\,\mathbf{j} + \frac{1}{t+1}\,\mathbf{k}$ for any time t. Find the particle's acceleration and postition vectors $\mathbf{a}(t)$ and $\mathbf{r}(t)$ at any time t. [20 pts]

Please put problem 4 on answer sheet 4

4. (a) Show that the line with parametrization $\mathbf{r}(t) = (2+t)\,\mathbf{i} + (1+t)\,\mathbf{j} + \left(\frac{1}{2} - \frac{1}{2}t\right)\,\mathbf{k}$ does not intersect the plane $x + 2y + 6z = 10$. [10 pts]

   (b) Find the symmetric equation of the line containing the points $(1, 2, 0)$ and $(3, 6, 0)$. [10 pts]

Please put problem 5 on answer sheet 5

5. Find and categorize all local maximum and minimum values and saddle points of: [20 pts]

$$f(x, y) = 3xy - x^2y - xy^2$$

Turn Over!

Please put problem 6 on answer sheet 6

6. A mountain's altitude is given by $h(x, y) = 2000 - 0.01x^2 - 0.005y^2$. You are standing on the mountain at the point $(50, 100, 1925)$.

   (a) You begin to walk in the direction of $\mathbf{a} = 3\,\mathbf{i} - 2\,\mathbf{j}$. Will your altitude be increasing or decreasing? [10 pts]

   (b) In which direction should you move so that your altitude increases the most rapidly? What is this rate of change? [10 pts]

Please put problem 7 on answer sheet 7

7. (a) Find the volume of the solid enclosed by the cylinder $x^2 + y^2 = 4$ and between the planes $y + z = 2$ and $z = 0$. [12 pts]

   (b) Let D be the solid between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant. Parametrize the following integral as an iterated integral in spherical coordinates. **Do not evaluate this integral.** [8 pts]

$$\iiint_D \sqrt{x^2 + y^2 + z^2}\, dV$$

Please put problem 8 on answer sheet 8

8. Evaluate the integral: [20 pts]

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3 + 1}\, dy\, dx$$

Please put problem 9 on answer sheet 9

9. Let C be the intersection of the cylinder $(x-1)^2 + y^2 = 1$ with the plane $x + y + z = 4$ and with counterclockwise orientation when viewed from above. Use Stokes' Theorem to convert the line integral [20 pts]

$$\int_C xy\ dx + y\ dy + xz\ dz$$

to a surface integral. Parametrize the surface integral as an iterated integral in polar coordinates. **Do not evaluate this integral.**

Please put problem 10 on answer sheet 10

10. (a) Use the Fundamental Theorem of Line Integrals to evaluate $\int_C 2x\ dx - z\ dy + (1-y)\ dz$ where C is the curve with parametrization $\mathbf{r}(t) = \sqrt{t+1}\,\mathbf{i} + \frac{1}{t+1}\,\mathbf{j} + e^t\,\mathbf{k}$ for $0 \le t \le 3$. [10 pts]

    (b) Use the Divergence Theorem to evaluate $\iint_\Sigma (2x\,\mathbf{i} + 3y\,\mathbf{j} + 3z\,\mathbf{k}) \cdot \mathbf{n}\ dS$ where Σ is the sphere $x^2 + y^2 + z^2 = 5$ oriented inwards. That is, the unit normal vector $\mathbf{n}$ on Σ is directed inwards. [10 pts]

Welcome to the End of the Exam