Math 241 Fall 2018 Final Exam

- Be sure your name, section number and problem number are on each answer sheet and that you have copied and signed the honor pledge on the first answer sheet.
- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets but if you do so, please write "See Back" or something similar on the bottom of the front so we know!
- No calculators or formula sheets are permitted.
- For problems with multiple parts, whether the parts are related or not, be sure to go on to subsequent parts even if there is some part you cannot do.
- Simplification of answers is not necessary unless specified. Please leave answers such as $5\sqrt{2}$ or 3π in terms of radicals and π and *do not convert to decimals*.

Please put problem 1 on answer sheet 1

1. Parts (a) and (b) are independent.

   (a) Use the dot product to show that the three points $A = (-2, 3, -4), B = (0, 11, -1)$, and $C = (1, 10, -5)$ form a right triangle. Then find the length of the hypotenuse. [10 pts]

   (b) Find the distance between the point $Q = (-2, 1, 3)$ and the line $x = 2$, $\frac{y+3}{-2} = z - 4$. [10 pts]

Please put problem 2 on answer sheet 2

2. Parts (a) and (b) are independent.

   (a) An 80 pound force and a 50 pound force are applied to an object at the same point with an angle of $\frac{\pi}{6}$ between them. Find the magnitude of the resultant force on the object. [10 pts]

   (b) Show that the two lines $\mathbf{r}_1(t) = (t+1)\,\mathbf{i} + 2t\,\mathbf{j} + 3\,\mathbf{k}$ and $\mathbf{r}_2(s) = s\,\mathbf{i} + (4-s)\,\mathbf{j} + (s+1)\,\mathbf{k}$ intersect and are not parallel, then find the equation of the plane containing them. [10 pts]

Please put problem 3 on answer sheet 3

3. Consider the curve C parametrized by $\mathbf{r}(t) = e^t \cos t\,\mathbf{i} + e^t \sin t\,\mathbf{j}$.

   (a) Find the tangent vector $\mathbf{T}(t)$. Simplify. [10 pts]

   (b) Find the normal vector $\mathbf{N}(t)$. [10 pts]

Please put problem 4 on answer sheet 4

4. Let C be the curve defined as the portion of the parabola $y = x^2$ in the plane $z = -2$ between the points $(2, 4, -2)$ and $(3, 9, -2)$.

   (a) Find a parametrization of C. [10 pts]

   (b) Set up the definite integral that computes the length of C. **Do not evaluate the integral!** [10 pts]

Turn Over!

Please put problem 5 on answer sheet 5

5. Use Lagrange Multipliers to find the extreme values of $f(x, y) = 3x - y$ subject to the constraint $x^2 + 2y^2 = 1$. (You may assume these extreme values exist.) [20 pts]

Please put problem 6 on answer sheet 6

6. Consider $f(x, y) = x^2y + xy + y^3$. Use the tangent plane approximation at $(1, 2)$ to approximate $f(0.95, 2.1)$. [20 pts]

Please put problem 7 on answer sheet 7

7. Let D be the solid that lies inside the sphere $x^2+y^2+z^2 = 2$ and outside the cylinder $x^2+y^2 = 1$.

   (a) Set up an iterated triple integral in spherical coordinates for the volume of D. [10 pts]
   Do not evaluate the integral!

   (b) Set up an iterated triple integral in cylindrical coordinates for $\iiint\limits_D x^2\,dV$. [10 pts]
   Do not evaluate the integral!

Please put problem 8 on answer sheet 8

8. Let R be the region enclosed by the ellipse given by $9x^2 + y^2 = 9$. Using an appropriate change of variables, evaluate $\iint\limits_R x^2\ dA$. Make sure you specify the change of variables, and draw the new region. **Evaluate the integral!** [20 pts]

Please put problem 9 on answer sheet 9

9. Let Σ be the portion of the plane $z = 9-x$ inside the cylinder $r = 2\cos\theta$. Let C be the boundary of Σ with counterclockwise orientation when viewed from above. Use Stokes' Theorem to rewrite the integral $\int_C 2x\,dx + yz\,dy + x^2z^2\,dz$ as a surface integral and then proceed until you have an iterated double integral. **Do not evaluate the integral!** [20 pts]

Please put problem 10 on answer sheet 10

10. Parts (a) and (b) are independent.

    (a) Use Green's Theorem to evaluate $\int_C xy\,dx + x\,dy$ where C is the triangle with vertices $(0, 0)$, $(3, 0)$, and $(3, 6)$, oriented clockwise. **Evaluate the integral!** [10 pts]

    (b) Let Σ be the portion of the paraboloid $z = 16 - x^2 - y^2$ restricted by $0 \le x \le 2$ and $0 \le y \le 3$. Write down an iterated double integral for the surface area of Σ. [10 pts]
    Do not evaluate the integral!

Welcome to the End of the Exam