Math 241 Spring 2018 Final Exam

- Be sure your name, section number and problem number are on each answer sheet and that you have copied and signed the honor pledge on the first answer sheet.
- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets but if you do so, please write "See Back" or something similar on the bottom of the front so we know!
- No calculators or formula sheets are permitted.
- For problems with multiple parts, whether the parts are related or not, be sure to go on to subsequent parts even if there is some part you cannot do.
- Please leave answers such as $5\sqrt{2}$ or 3π in terms of radicals and π and *do not convert to decimals.*

Please put problem 1 on answer sheet 1

1. Parts (a) and (b) are independent.

   (a) Let $P = (-1, 3)$, $Q = (-2, 1)$, and $R = (1, -4)$. Find the projection of $\overrightarrow{PQ}$ onto $\overrightarrow{QR}$. [10 pts]

   (b) Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied to an object located at the origin in the xy-plane. The force $\mathbf{F}_1$ has a magnitude of 60 and makes an angle of $\frac{\pi}{6}$ with the positive x-axis and force $\mathbf{F}_2$ has a magnitude of 100 and makes an angle of $\frac{3\pi}{4}$ with the positive x-axis. Find the magnitude and direction (angle from positive x-axis) of the sum of these forces. [10 pts]

Please put problem 2 on answer sheet 2

2. Parts (a) and (b) are independent.

   (a) Find the distance from the point $(1, -3, 2)$ to the line containing the points $(1, 0, 1)$ and $(5, 2, 0)$. [10 pts]

   (b) Find an equation of the form $ax + by + cz = d$ of the plane that contains the point $(1, 2, 3)$ and the line [10 pts]
   $$\frac{x}{2} = \frac{y+3}{7} = \frac{z-4}{5}.$$

Please put problem 3 on answer sheet 3

3. Parts (a) and (b) are independent.

   (a) Find a parametrization of the quarter circle in the plane $z = 1$ with endpoints $(-1, 0, 1)$ and $(0, -1, 1)$ and center $(0, 0, 1)$. [8 pts]

   (b) Determine if the following pair of lines is parallel, intersecting, or neither. If the lines intersect, find the point at which they intersect. [12 pts]
   $$\mathbf{r}_1(t) = (1 + 6t)\mathbf{i} + (3 - 7t)\mathbf{j} + (2 + t)\mathbf{k}$$
   $$\mathbf{r}_2(s) = (10 + 3s)\mathbf{i} + (6 + s)\mathbf{j} + (14 + 4s)\mathbf{k}$$

Please put problem 4 on answer sheet 4

4. Find all critical points of $f(x, y) = x^3 - 6x^2 - 5y^2$ and classify each as a relative minimum, maximum, or saddle point. [20 pts]

Turn Over!

Please put problem 5 on answer sheet 5

5. Use Lagrange Multipliers to find the maximum and minimum of $f(x, y) = x^3 - 6x^2 - 5y^2$ subject to the constraint $x^2 + y^2 = 1$. [20 pts]
   Note: Your system will have six solutions.

Please put problem 6 on answer sheet 6

6. Let R be the region in the first quadrant of the xy-plane bounded by the lines $y = x$ and $y = 3x$ and by the hyperbolas $xy = 1$ and $xy = 5$. Use the change of variables $x = \frac{u}{v}$ and $y = v$ to set up an iterated double integral in the uv-plane representing the area of R. [20 pts]
   Do not evaluate this integral.

Please put problem 7 on answer sheet 7

7. Parts (a) and (b) are independent.
   (a) Find the unit vector direction of maximum increase of $f(x, y) = x^2y - xy$ at the point $(2, -3)$. [10 pts]
   (b) Use tangent plane approximation to approximate $(16.05)^{1/4}(7.95)^{2/3}$. [10 pts]

Please put problem 8 on answer sheet 8

8. Write down the iterated triple integral representing the volume of the solid region D bounded between the spheres (centered at the origin) of radius 1 and 3, and the upper part of the cone $z^2 = 3(x^2 + y^2)$. [15 pts]
   Do not evaluate this integral.

Please put problem 9 on answer sheet 9

9. Let C be the intersection of the parabolic sheet $y = 16 - x^2$ with the cylinder $x^2 + z^2 = 4$ with counterclockwise orientation when viewed from the positive y-axis. Use Stokes' Theorem to convert the line integral [25 pts]
$$\int_C x^2z\,dx + x\,dy + yz\,dz$$
   to a surface integral. Parametrize the surface to obtain a double iterated integral.
   Do not evaluate this integral.

Please put problem 10 on answer sheet 10

10. Parts (a) and (b) are independent.
    (a) Evaluate $\int_C (2x + y)\,dx + x\,dy$ where C is the line segment joining $(1, 4)$ to $(5, -1)$. [10 pts]
    (b) Evaluate $\int\int_\Sigma (2x\,\mathbf{i} + 5z\,\mathbf{j} - 7y\,\mathbf{k}) \cdot \mathbf{n}\,dS$ where Σ is the sphere $x^2 + y^2 + z^2 = 9$ with inwards orientation. [10 pts]

Welcome to the End of the Exam