Math 241 Spring 2019 Final Exam

- Be sure your name, section number and problem number are on each answer sheet and that you have copied and signed the honor pledge on the first answer sheet.
- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets but if you do so, please write "See Back" or something similar on the bottom of the front so we know!
- No calculators or formula sheets are permitted.
- For problems with multiple parts, whether the parts are related or not, be sure to go on to subsequent parts even if there is some part you cannot do.
- Simplification of answers is not necessary. Please leave answers such as $5\sqrt{2}$ or 3π in terms of radicals and π and *do not convert to decimals.*

Please put problem 1 on answer sheet 1

1. Parts (a) and (b) are independent.

   (a) Find parametric equations for the line L containing the points $(-2, 0, 1)$ and $(4, -2, -3)$. [10 pts]

   (b) Do the planes $\mathcal{P}_0 : 2x - y + 3z = -2$, and $\mathcal{P}_1 : -2x - 3y + z = 6$ intersect? If so, find symmetric equations for the line of intersection. If not, explain why not. [10 pts]

Please put problem 2 on answer sheet 2

2. Parts (a) and (b) are independent.

   (a) Find an equation for the plane containing the points $P = (1, -3, 1)$, $Q = (2, 2, 0)$, and $R = (-4, -1, 1)$. [10 pts]

   (b) Find the distance between the point $S = (-2, 3, 1)$ and the plane $-4x + y - 2z = 0$. [10 pts]

Please put problem 3 on answer sheet 3

3. Parts (a) and (b) are independent.

   (a) Compute the length of the curve C_1 with parametrization: [10 pts]

   $$\mathbf{r}(t) = \tfrac{1}{3}(1+t)^{\frac{3}{2}}\,\mathbf{i} + \tfrac{1}{3}(1-t)^{\frac{3}{2}}\,\mathbf{j} + t\sqrt{3}\,\mathbf{k} \text{ for } -\tfrac{1}{4} \le t \le \tfrac{1}{2}$$

   (b) Find all points (if any) where the curve C_2 with the following parametrization meets the sphere of radius 3 centered at the origin: [10 pts]

   $$\mathbf{r}(t) = \sqrt{t}\,\mathbf{i} + \sqrt{t+1}\,\mathbf{j} + t\,\mathbf{k} \text{ for } t \ge 0$$

Please put problem 4 on answer sheet 4

4. Consider the curve parameterized by $\mathbf{r}(t) = \cos^3 t\,\mathbf{i} + \sin^3 t\,\mathbf{j}$.

   (a) Find the tangent vector $\mathbf{T}(t)$. [12 pts]

   (b) Find the normal vector $\mathbf{N}(t)$. [8 pts]

Turn Over!

Please put problem 5 on answer sheet 5

5. Let $f(x, y, z) = z^2 - 8\sqrt{x^2 - 3y}$.

   (a) Find $\text{grad} f$. [5 pts]

   (b) Find an equation of the plane tangent to the level surface for f at $(5, 3, 2)$. [5 pts]

   (c) Find $D_{\mathbf{u}}f$ at $(5, 3, 2)$ where $\mathbf{u}$ is pointing in the direction $1\,\mathbf{i} + 2\,\mathbf{j} - 3\,\mathbf{k}$. [5 pts]

   (d) Find the smallest value of $D_{\mathbf{u}}f$ at $(5, 3, 2)$. [5 pts]

Please put problem 6 on answer sheet 6

6. Let $f(x, y) = x^4 + y^2$.

   (a) Use Lagrange Multipliers to find the maximum and minimum of $f(x, y)$ subject to the constraint $x^2 + y^2 = 1$. [15 pts]

   (b) Find the maximum and minimum of $f(x, y)$ subject to the constraint $x^2 + y^2 \leq 1$. [5 pts]

Please put problem 7 on answer sheet 7

7. Use the change of variables $u = y - x$ and $v = y + x$ to evaluate the double integral: [20 pts]

$$\iint_R (y - x) \sin\left((y + x)^3\right)\, dA$$

   where R is the triangle with vertices $(0, 0)$, $(2, 2)$ and $(0, 4)$.

Please put problem 8 on answer sheet 8

8. Find the volume of the solid region bounded above by the sphere $x^2 + y^2 + z^2 = 8$ and below by the paraboloid $2z = x^2 + y^2$. [20 pts]

Please put problem 9 on answer sheet 9

9. Parts (a) and (b) are independent.

   (a) Let Σ be the portion of the cylinder $x^2 + y^2 = 9$ between $z = 1$ and $z = 8$. If the mass density at (x, y, z) is given by $f(x, y, z) = x^2 z$ write down an iterated double integral for the mass of Σ. [12 pts]
   Do not evaluate the integral!

   (b) Let C be the triangle with vertices $(0, 4)$, $(2, 0)$ and $(2, 4)$ with clockwise orientation. Use Green's Theorem to evaluate: [8 pts]

$$\int_C 4y\, dx + 9x\, dy$$

Please put problem 10 on answer sheet 10

10. Let Σ be the portion of the plane $x + 2y + z = 10$ in the first octant. Let C be the boundary of Σ with counterclockwise orientation when viewed from above. Use Stokes' Theorem to rewrite the integral $\int_C 3xy\, dx + z^2\, dy + xy\, dz$ as a surface integral and then proceed until you have an iterated double integral. **Do not evaluate the integral!** [20 pts]

Welcome to the End of the Exam