Math 241 Final Spring 2020 **Justin Wyss-Gallifent**

Directions: Do not simplify non-Calc3 things unless indicated. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words, pictures or ideas which are relevant to the problem. You may use any resources except asking other people. Answers should be neat and tidy and complete.

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1. Let $\mathcal{L}$ be the line with symmetric equation:

$$\frac{x-1}{2} = \frac{z}{3}\ ,\ y = 2$$

Let $\mathcal{P}$ be the plane with equation:

$$2x + y - 3z = 1$$

   (a) Find the point at which $\mathcal{L}$ meets $\mathcal{P}$. [10 pts]

   (b) At this point, what is the cosine of the angle between $\mathcal{L}$ and the normal vector for $\mathcal{P}$? [15 pts]

   (c) If you went to the point on the line where $x = 7$ and drew a sphere centered at that point, how large could the radius be before the sphere hit the plane? [15 pts]

2. Define the function:

$$f(x,y) = xy^2 + 2xy - y$$

   (a) Find the equation of the plane tangent to the graph of $f(x,y)$ at $(1,2)$. Write this in the form $ax + by + cz = d$. [25 pts]

   (b) Calculate $\int_C \nabla f(x,y) \cdot d\mathbf{r}$ where C is the curve parametrized by $\mathbf{r}(t) = (t^2+t)\,\mathbf{i} + (5t+2)\,\mathbf{j}$ for $1 \le t \le 2$. [15 pts]

3. Let C be the counterclockwise curve consisting of the semicircle $x^2 + y^2 = 9$ with $x \ge 0$ along with the line segment joining the endpoints of that semicircle. Consider the integral:

$$\int_C 6y\,dx + 15x\,dy$$

   (a) Parametrize C (you'll need two parametrizations) and use them to evaluate the integral. [20 pts]

   (b) Use Green's Theorem to rewrite the line integral as an integral over a region R. Evaluate this integral. [15 pts]

   (c) Your answers to (a) and (b) should be equal. Are they? Yes or no is enough. [5 pts]

4. Let C be the intersection of the plane $z = 9 - y$ with the cylinder $x^2 + y^2 = 4$ with counterclockwise orientation when viewed from above. Consider the integral:

$$\int_C x\,dx + x\,dy + z\,dz$$

   (a) Parametrize C and use this parametrization to evaluate the integral. [15 pts]

   (b) Use Stokes' Theorem to rewrite the line integral as a surface integral over an appropriate surface. Evaluate this surface integral. [15 pts]

   (c) Write down the Matlab command you would use to evaluate the final iterated integral in (b). [5 pts]

   (d) Your answers to (a) and (b) should be equal. Are they? Yes or no is enough. [5 pts]

5. Define the function:

$$f(x,y) = x^3 + y^3 + 3xy$$

   (a) Find and categorize all critical points of $f(x,y)$. [20 pts]

   (b) Without doing any integration explain how you know that $\int_C f(x,y)\,ds > 25$ where C is the part of the circle $x^2 + y^2 = 4$ in the first quadrant. [20 pts]
   Hint: This is tricky. The integral measures mass. Can you find a constant A such that $f(x,y) \ge A$ on C?