Math 241 Fall 2021 Final Exam

- Be sure your name, section number and problem number are on each answer sheet and that you have copied and signed the honor pledge on the first answer sheet.
- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets but if you do so, please write "See Back" or something similar on the bottom of the front so we know!
- No calculators or formula sheets are permitted.
- For problems with multiple parts, whether the parts are related or not, be sure to go on to subsequent parts even if there is some part you cannot do.
- Simplification of answers is not necessary. Please leave answers such as $5\sqrt{2}$ or 3π in terms of radicals and π and *do not convert to decimals.*

Please put problem 1 on answer sheet 1

1. (a) Find the projection of the vector $\mathbf{b} = 1\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$ onto the vector $\mathbf{a} = 2\hat{\imath} - 1\hat{\jmath} + 2\hat{k}$. [10 pts]

   (b) Find symmetric equations for the line that contains the point $(-3, -3, 1)$ and is perpendicular to the plane $2x - 3y + 4z = 7$. [10 pts]

Please put problem 2 on answer sheet 2

2. (a) Find an equation for the plane that contains both the lines $x = 1 + 2t, y = t, z = 4 + 4t$, and $x = 1 - t, y = 2t, z = 4 + 3t$. Write this in the form $ax + by + cz = d$. [10 pts]

   (b) Suppose the points $(1, 2, 3)$ and $(11, 10, -5)$ are on opposite sides of a sphere (the center of the sphere is the midpoint of the two points). Find the equation of the sphere. [10 pts]

Please put problem 3 on answer sheet 3

3. A particle with initial position $\mathbf{r}(0) = 0\hat{\imath} + 0\hat{\jmath} + 1\hat{k}$ has velocity $\mathbf{v}(t) = t\hat{\imath} + 2t^2\hat{\jmath} + \sqrt{t}\hat{k}$ for any time $t \geq 0$.

   (a) Find the particle's position vector $\mathbf{r}(t)$ at any time $t \geq 0$. [10 pts]

   (b) Find the tangential component of acceleration at $t = 1$. [10 pts]

Please put problem 4 on answer sheet 4

4. Let $\mathbf{r}(t) = \sin(t)\hat{\imath} + \cos(t)\hat{\jmath} + t^{3/2}\hat{k}$ for $\frac{\pi}{6} \leq t \leq \frac{\pi}{2}$ be the parameterization of a curve.

   (a) Determine whether the parameterization is smooth, piecewise smooth or neither. [5 pts]

   (b) Find the length of the curve. [15 pts]

Please put problem 5 on answer sheet 5

5. Use the method of Lagrange multipliers to find the point on the ellipse $2x^2 + xy + y^2 = 7$ with the largest possible x-coordinate. [20 pts]

Turn Over!

Please put problem 6 on answer sheet 6

6. Let $f(x, y, z) = x^3 + 3xy + xe^z$, and suppose $x(t)$, $y(t)$, and $z(t)$ are functions satisfying $x(0) = 1$, $y(0) = 0$, $z(0) = 0$, $x'(0) = 1$, $y'(0) = 2$, and $z'(0) = 3$. Calculate the following: [20 pts]

$$\frac{df}{dt} \text{ at } t = 0$$

Please put problem 7 on answer sheet 7 ‘

7. Reverse the order of integration and evaluate the following integral: [20 pts]

$$\int_0^{\sqrt{\pi}/2} \int_y^{\sqrt{\pi}/2} \cos(x^2)\,dx\,dy$$

Please put problem 8 on answer sheet 8

8. Let D be the solid above the xy-plane, below the cone $z = \sqrt{3x^2 + 3y^2}$, and between the spheres $x^2 + y^2 + z^2 = 16$ and $x^2 + y^2 + z^2 = 25$. Evaluate the following integral: [20 pts]

$$\iiint_D \frac{36\sqrt{x^2 + y^2 + z^2}}{\sqrt{x^2 + y^2}}\,dV$$

Please put problem 9 on answer sheet 9

9. (a) Let C be any curve from $(1, -2, 3)$ to $(0, 5, 2)$ not passing through $z = 0$. Evaluate the following integral: [10 pts]

$$\int_C 2x\,dx + \left(1 + \frac{1}{z}\right)\,dy - \frac{y}{z^2}\,dz$$

   (b) Let C be the counterclockwise curve consisting of the semicircle $x^2 + y^2 = 9$ with $x \geq 0$ along with the line segment joining the endpoints of that semicircle. Use Green's Theorem to evaluate the following integral: [10 pts]

$$\int_C 6y\,dx + 15x\,dy$$

Please put problem 10 on answer sheet 10

10. Let C be the intersection of the plane $z = 9 - y$ with the cylinder $x^2 + y^2 = 4$ with counter-clockwise orientation when viewed from above. Consider the integral: [20 pts]

$$\int_C z^2\,dx + x\,dy + z\,dz$$

Use Stokes' Theorem to rewrite the line integral as an iterated double integral. Do not evaluate this integral.

Welcome to the End of the Exam