MATH 241 Sections 03** Final Spring 2021

Exam Submission:

1. Submit this exam to Gradescope.
2. Tag your problems!
3. You may print the exam, write on it, scan and upload.
4. Or you may just write on it on a tablet and upload.
5. Or you are welcome to write the answers on a separate piece of paper if other options don't appeal to you, then scan and upload.

Exam Rules:

1. You may ask me for clarification on questions but you may not ask me for help on questions!
2. You are permitted to use any non-interactive resources. This includes books, static pages on the internet, your notes, and YouTube videos.
3. You are not permitted to use any interactive resources. This includes your friends, your friends' friends, your calculator, Matlab, Wolfram Alpha, and online chat groups.
   Exception: Calculators are fine for basic arithmetic.
4. If you are unsure about whether a resource is considered "interactive" simply ask me and I'll let you (and everyone) know.
5. Petting small animals for stress relief is acceptable and is not considered an "interactive resource".

Work Shown:

1. Show all work as appropriate for and using techniques learned in this course.
2. Any pictures, work and scribbles which are legible and relevant will be considered for partial credit.

1. Given the two vectors:

$$\bar{\boldsymbol{u}} = 2\,\hat{\imath} + 3\,\hat{\jmath} - 5\,\hat{k} \text{ and } \bar{\boldsymbol{v}} = 5\,\hat{\imath} + 0\,\hat{\jmath} - 7\,\hat{k}$$

(a) Find $\bar{\boldsymbol{u}} \times \bar{\boldsymbol{v}}$. [5 pts]
Solution:

(b) Find $\text{Proj}_{\bar{\boldsymbol{v}}}\bar{\boldsymbol{u}}$. [10 pts]
Solution:

(c) Find a vector of length 42 pointing in the same direction as $\bar{\boldsymbol{u}}$. [5 pts]
Solution:

2. Consider the plane that passes through the point $P = (2, 5, 1)$ and includes the line with symmetric equation:

$$\frac{x-1}{2} = z+3 \quad , \quad y = 1$$

(a) Find an equation for this plane in the form $ax + by + cz = d$ [20 pts]

Solution:

(b) Find y so that $(1, y, 3)$ is on this plane. [5 pts]

Solution:

3. An object follows the path with parametrization:

$$\bar{\boldsymbol{r}}(t) = \cos t\,\hat{\imath} + 0\,\hat{\jmath} + \sin t\,\hat{k} \quad \text{for} \quad 0 \leq t \leq 2\pi$$

Floating in space is the paraboloid with equation:

$$z = \frac{2}{3}(x^2 + y^2)$$

(a) The object hits the paraboloid twice. Which point is first and which point is second? [10 pts]

Solution:

(b) Find the distance that the object travels within the paraboloid. [5 pts]

Solution:

4. Let R be the region bounded by the lines $x = 2$, $y = x - 1$ and $y = 1 - x$. [25 pts] Use the change of variables given by $u = x + y$ and $v = x - y$ to evaluate the following integral:

$$\iint_R \frac{1}{x+y}\, dA$$

You Should Evaluate Your Resulting Integral!

Solution:

5. Define the function $f(x, y) = x^2y - 3xy^2 - y$.

(a) Find $\nabla f(1, 2)$ and simplify. [5 pts]
Solution:

(b) Explain why it is not possible to find a unit vector $\bar{\boldsymbol{u}}$ with $\bar{\boldsymbol{u}} \cdot \nabla f(1, 2) = 15$. [10 pts]
Solution:

(c) Explain why it is possible to find a unit vector $\bar{\boldsymbol{u}}$ with $\bar{\boldsymbol{u}} \cdot \nabla f(1, 2) = 3$. [10 pts]
Solution:

6. Find and categorize (as relative max, relative min, or saddle points) all five critical points for the function: [25 pts]

$$f(x, y) = x^2y^2 - x^2 - 4y^2$$

Solution:

7. Evaluate the integral: [20 pts]

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \sqrt{x^2+y^2}\, dy\, dx$$

You Should Evaluate Your Resulting Integral!

Solution:

8. Suppose Σ is the portion of the paraboloid $y = 4 - x^2$ in the first octant and below the plane $z = 3$. Let C be the edge of Σ with clockwise orientation when viewed from out in the first octant looking towards the origin. [25 pts]

Apply Stokes' Theorem to the integral:

$$\int_C (x + z)\, dx + x^2\, dy + xy\, dz$$

Proceed until you have an iterated double integral with the integrand simplified.

You Should Not Evaluate Your Resulting Integral!

Solution:

9. Define the vector field $\bar{\boldsymbol{F}}(x, y) = 4x\,\hat{\imath} + 6xy\,\hat{\jmath}$.

(a) If C is the counterclockwise triangle with vertices $(0,0)$, $(2,0)$, and $(0,4)$, calculate $\int_C \bar{\boldsymbol{F}}(x,y) \cdot d\bar{\boldsymbol{r}}$. [10 pts]

You Should Evaluate Your Resulting Integral!

Solution:

(b) If C is the line segment from $(0,0)$ to $(2,4)$, calculate $\int_C \bar{\boldsymbol{F}} \cdot d\bar{\boldsymbol{r}}$. [10 pts]

You Should Evaluate Your Resulting Integral!

Solution: