Math 241 Spring 2022 Final Exam

- Be sure your name, section number and problem number are on each answer sheet and that you have copied and signed the honor pledge on the first answer sheet.
- Follow the instructions as to which problem goes on which answer sheet. You may use the back of the answer sheets but if you do so, please write "See Back" or something similar on the bottom of the front so we know!
- No calculators or formula sheets are permitted.
- For problems with multiple parts, whether the parts are related or not, be sure to go on to subsequent parts even if there is some part you cannot do.
- Simplification of answers is not necessary. Please leave answers such as $5\sqrt{2}$ or 3π in terms of radicals and π and *do not convert to decimals.*

Please put problem 1 on answer sheet 1

1. (a) Suppose $\mathbf{u} = 3\,\hat{\boldsymbol{\imath}} + 2\,\hat{\boldsymbol{\jmath}} - 1\,\hat{\boldsymbol{k}}$ and $\mathbf{v} = 0\,\hat{\boldsymbol{\imath}} + 5\,\hat{\boldsymbol{\jmath}} + 7\,\hat{\boldsymbol{k}}$. Simplify the single expression: [15 pts]

$$(\mathbf{u} \cdot \mathbf{v})(\mathbf{u} \times \mathbf{v})$$

   (b) Write down a parameterization $\mathbf{r}(t)$ of the circle of radius 2 in the plane $z = 5$ and centered at $(1, 3, 5)$. [10 pts]

Please put problem 2 on answer sheet 2

2. (a) Find the directional derivative of the function $f(x, y) = x^2 y + \frac{x}{y}$ at the point $(1, 2)$ in the direction of the vector $5\hat{\boldsymbol{\imath}} - 3\hat{\boldsymbol{\jmath}}$. [10 pts]

   (b) Find the equation of the plane tangent to the graph of the function $f(x, y) = x^2 y + x - y$ at the point $(1, 2, 1)$. [15 pts]

Please put problem 3 on answer sheet 3

3. (a) Find the distance between the two parallel lines $\mathcal{L}_1$ and $\mathcal{L}_2$ given with symmetric equations: [10 pts]

$$\mathcal{L}_1 : \quad x = \frac{y}{2} = \frac{z}{3}$$
$$\mathcal{L}_2 : \quad x + 1 = \frac{y - 1}{2} = \frac{z + 5}{3}$$

   (b) Suppose $\theta(t)$ is a function satisfying: [15 pts]

$$\theta(1) = \frac{\pi}{6} \quad \text{and} \quad \theta'(1) = 2$$

   Find the tangent $\mathbf{T}$ vector at time $t = 1$ to the curve with parameterization:

$$\mathbf{r}(t) = \cos(\theta(t))\,\hat{\boldsymbol{\imath}} + \sin(\theta(t))\,\hat{\boldsymbol{\jmath}} + \theta(t)\,\hat{\boldsymbol{k}}$$

Please put problem 4 on answer sheet 4

4. Find and categorize all critical points for the function: [25 pts]

$$f(x, y) = x^2 y + 4xy - 12y^2$$

Exam Continues on Other Side

Please put problem 5 on answer sheet 5

5. (a) Use the Fundamental Theorem of Line Integrals to evaluate: [10 pts]

$$\int_C (2x + y^2)\, dx + 2xy\, dy$$

Where C is the line segment from $(1, 3)$ to $(100, 200)$.

(b) Consider the solid D that lies above the cone $z = \sqrt{3x^2 + 3y^2}$ and between the two spheres $x^2 + y^2 + z^2 = 2$ and $x^2 + y^2 + z^2 = 3$. Assume the mass density at each point (x, y, z) is given by the distance of each point to the origin. Write down an iterated triple integral for the mass of D. [15 pts]
You Should Not Evaluate Your Resulting Integral!

Please put problem 6 on answer sheet 6

6. Consider the region R in the xy-plane bounded by the circle $x^2 + y^2 = 2x$.

(a) Treating R as a vertically simple region, set up an iterated double integral in rectangular coordinates that is equal to $\iint_R (x + y)\, dA$. [12 pts]
You Should Not Evaluate Your Resulting Integral!

(b) Set up an iterated double integral in polar coordinates that is equal to $\iint_R (x + y)\, dA$. [13 pts]
You Should Not Evaluate Your Resulting Integral!

Please put problem 7 on answer sheet 7

7. Let Σ be the part of the vertical plane $x + 2y = 4$ in the first octant and below $z = 5$. Let C be the boundary/edge of Σ with clockwise orientation when viewed from the origin. Consider the integral: [25 pts]

$$\int_C z^2\, dx + x\, dy + z\, dz$$

Use Stokes' Theorem to rewrite the line integral as an iterated double integral.
You Should Not Evaluate Your Resulting Integral!

Please put problem 8 on answer sheet 8

8. (a) Let D be the solid object above the xy-plane and below the paraboloid $z = 4 - x^2 - y^2$. Let Σ be the surface of D, oriented inwards. Evaluate the following integral: [15 pts]

$$\iint_\Sigma (2x\hat{\boldsymbol{\imath}} + 5x\hat{\boldsymbol{\jmath}} + 7z\hat{\boldsymbol{k}}) \cdot \mathbf{n}\, dS$$

(b) Let C be the triangle with vertices $(0, 0)$, $(5, 1)$, and $(3, 1)$ with clockwise orientation. Use Green's Theorem to evaluate the following integral: [10 pts]

$$\int_C 6y\, dx + 15x\, dy$$

Welcome to the End of the Exam