MATH 241 Sections 03** Exam 1

Exam Submission:

1. Submit this exam to Gradescope.
2. Tag your problems!
3. You may print the exam, write on it, scan and upload.
4. Or you may just write on it on a tablet and upload.
5. Or you are welcome to write the answers on a separate piece of paper if other options don't appeal to you, then scan and upload.

Exam Rules:

1. You may ask me for clarification on questions but you may not ask me for help on questions!
2. You are permitted to use your notes and the textbook.
3. You are not permitted to use other resources. Thus no friends, internet, etc. Exception: Calculators are fine for basic arithmetic.

Work Shown:

1. Show all work as appropriate for and using techniques learned in this course.
2. Any pictures, work and scribbles which are legible and relevant will be considered for partial credit.

1. Given the two vectors: [10 pts]

$$\bar{a} = 1\,\hat{\imath} + 8\,\hat{\jmath} + 4\,\hat{k}$$
$$\bar{b} = 4\,\hat{\imath} + 2\,\hat{\jmath} - 2\,\hat{k}$$

Rewrite $\bar{a}$ as $\bar{a} = \bar{a}_1 + \bar{a}_2$ where $\bar{a}_1$ is parallel to $\bar{b}$ and $\bar{a}_2$ is perpendicular to $\bar{b}$. In addition, verify that your final $\bar{a}_1$ and $\bar{a}_2$ are in fact perpendicular.

2. Show that the set of points whose distance to $(-10, -5, 0)$ is three times their distance to $(6, 3, 0)$ form a sphere. Write the equation of this sphere in standard sphere form: [15 pts]

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2$$

In addition, draw a reasonable sketch of the sphere.

3. Find the equation of the plane containing the point $(7, -2, 10)$ and containing the line with parametric equations [10 pts]

$$\begin{aligned} x &= 2 + 3t \\ y &= -2t \\ z &= 6 + t \end{aligned}$$

Write this plane in the form $ax + by + cz = d$.

4. Given the two planes: [15 pts]

$$\mathcal{P}_1 : 2x - y + 4z = 10$$
$$\mathcal{P}_2 : -2x + 5y + z = 6$$

Find the symmetric equations of the line where the planes $\mathcal{P}_1$ and $\mathcal{P}_2$ intersect.

5. Sketch the three parametrizations given here. Indicate the start and end points with their coordinates and indicate the direction of the curve.

(a) $\bar{r}_1(t) = (1 + 4\cos t)\,\hat{\imath} + 3\sin t\,\hat{\jmath}$ [5 pts]
For: $0 \leq t \leq \frac{3\pi}{2}$
In 2D

(b) $\bar{r}_2(t) = (1 + t)\,\hat{\imath} + (2 - 3t)\,\hat{\jmath}$ [5 pts]
For: $1 \leq t \leq 3$
In 2D

(c) $\bar{r}_3(t) = |t|\,\hat{\imath} + 0\,\hat{\jmath} + t\,\hat{k}$ [5 pts]
For: $-1 \leq t \leq 2$
In 3D

6. Consider two curves with parametrizations: [10 pts]

$$\bar{r}_1(s) = (s^2 + 1)\,\hat{\imath} + (1 - s)\,\hat{\jmath} + (2 + 3s)\,\hat{k}$$
$$\bar{r}_2(t) = (2 + 15t)\,\hat{\imath} - 3t\,\hat{\jmath} + (5 + 9t)\,\hat{k}$$

These curves meet at two points. Find these points.

7. Given the parametrization $\bar{r}(t) = t\,\hat{\imath} + t^2\,\hat{\jmath} + t\,\hat{k}$,

(a) Find $\bar{v}(2)$ and $\bar{a}(2)$. [5 pts]

(b) Find $\bar{T}(2)$. [5 pts]

(c) Find the tangential and normal components of acceleration, a_T and a_N, at $t = 2$. [5 pts]

(d) Using the fact that $\bar{a} = a_T\bar{T} + a_N\bar{N}$, find $\bar{N}(2)$. [5 pts]

(e) Write down the integral which represents the distance along the curve between $t = 0$ and $t = 2$. Without calculating this integral explain how you know the value is at least $2\sqrt{6}$. [5 pts]