NAME (Neatly):

UID (Neatly):

Instructions:

1. Please do all problems on the pages and in the spaces provided. This exam will be scanned into Gradescope and if your answers are not in the correct locations they will not be found or graded!
2. Only simplify Calculus 3 related calculations.

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(Except for doodling for stress relief.)

1. Given the vectors:

$$\bar{\boldsymbol{a}} = 2\hat{\boldsymbol{\imath}} + 6\hat{\boldsymbol{\jmath}} - 4\hat{\boldsymbol{k}}$$
$$\bar{\boldsymbol{b}} = 0\hat{\boldsymbol{\imath}} - 3\hat{\boldsymbol{\jmath}} + 6\hat{\boldsymbol{k}}$$
$$\bar{\boldsymbol{c}} = 3\hat{\boldsymbol{\imath}} + \alpha\hat{\boldsymbol{\jmath}} + \alpha\hat{\boldsymbol{k}}$$

(a) Calculate $\text{Proj}_{\bar{\boldsymbol{b}}}\bar{\boldsymbol{a}}$. [5 pts]
Solution:

(b) Calculate all possible α so that $\bar{\boldsymbol{a}}$ and $\bar{\boldsymbol{c}}$ are perpendicular. [5 pts]
Solution:

2. Find the distance between the point $(1, 2, 3)$ and the plane $2x + 3y - 2z = 10$. Simplify. [10 pts]

Solution:

3. Find $a_T(1)$ for the curve with parameterization: [10 pts]

$$\bar{\boldsymbol{r}}(t) = t^3\hat{\boldsymbol{\imath}} + 5t\hat{\boldsymbol{\jmath}} + e^{2t-2}\hat{\boldsymbol{k}}$$

Solution:

4. Suppose an object follows the path given by the parameterization:

$$\bar{\boldsymbol{r}}(t) = (3t-1)\hat{\boldsymbol{\imath}} + 2t\hat{\boldsymbol{\jmath}} + t\hat{\boldsymbol{k}}$$

And given the plane with equation:

$$x + 2y - 3z = 10$$

(a) At what time does the object hit the plane? [5 pts]
Solution:

(b) At that instant, what is its velocity? [10 pts]
Solution:

5. Given the parameterization:

$$\bar{\boldsymbol{r}}(t) = \sin(\pi t)\hat{\boldsymbol{\imath}} + t^2\hat{\boldsymbol{\jmath}} + 5\hat{\boldsymbol{k}} \quad \text{with} \quad -1 \leq t \leq 1$$

(a) Is the parameterization closed? Justify. [5 pts]

Solution:

(b) Is the parameterization smooth? Justify. [10 pts]

Solution:

6. Find the symmetric equation of the line through the two points $(1, 4, -5)$ and $(8, 4, 10)$. [10 pts]

Solution:

7. Find the equation of the plane containing the origin and the line with symmetric equation: [15 pts]

$$\frac{x-1}{2} = \frac{5-y}{6} = z$$

Solution:

8. Plot each of the following in 3D. On each, mark at least one point with its coordinates.

(a) $x^2 + y^2 + (z-2)^2 = 4$ [5 pts]

Solution:

(b) $y = 3$ [5 pts]

Solution:

(c) $\bar{\boldsymbol{r}}(t) = \cos t\hat{\boldsymbol{\imath}} + \sin t\hat{\boldsymbol{\jmath}} + 2\hat{\boldsymbol{k}}$ for $0 \le t \le \pi$ [5 pts]

Solution: