Your Name NEATLY:

MATH241 Exam 1 Spring 2022 (Justin Wyss-Gallifent)

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.
Please: Do all work in the spaces provided!

1. Suppose the points $(1,2,3)$ and $(3,8,7)$ are on opposite sides of a sphere. Write down the equation of the sphere. [15 pts]

   Solution:

2. Suppose you have two nonzero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ and you calculate that $\text{Proj}_{\boldsymbol{b}}\boldsymbol{a} = \boldsymbol{a}$. What does this tell you about $\boldsymbol{a}$ and $\boldsymbol{b}$? Your answer should be a single simple sentence, no math or justification is required. [5 pts]

   Solution:

3. Suppose $\boldsymbol{a} = -2\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}} + \alpha\hat{\boldsymbol{k}}$ and $\boldsymbol{b} = 5\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} + \alpha\hat{\boldsymbol{k}}$ where α is unknown. What could α be in order for $\boldsymbol{a}$ and $\boldsymbol{b}$ to be perpendicular? [5 pts]

   Solution:

4. Calculate the distance from the point $(1, 4, 10)$ to the plane $x + 2y - 4z = 10$. [13 pts]

Solution:

5. Given the parameterization $\boldsymbol{r}(t) = \frac{1}{12}t^4\hat{\boldsymbol{\imath}} + \sqrt{t}\hat{\boldsymbol{\jmath}} + (t^2 + t)\hat{\boldsymbol{k}}$. Calculate $a_{\boldsymbol{N}}(4)$. [12 pts]

Solution:

6. Write down a closed parameterization for the circle in the plane $x = 5$ centered at $(5, 0, 0)$ with radius 3. No sketch is required! [10 pts]

Solution:

7. Find the parameterization $\boldsymbol{r}(t) = ...$ of the line passing through the point $(3, 2, 1)$ and parallel to the line with symmetric equation: [15 pts]

$$-1 - x = \frac{y - 4}{10}\,,\ z = 4$$

Solution:

8. Sketch the 2D curve with parameterization $\boldsymbol{r}(t) = t^2\hat{\boldsymbol{\imath}} + t\hat{\boldsymbol{\jmath}}$ for $-1 \leq t \leq 2$. Mark the start and end points with their coordinates. [10 pts]

Solution:

9. Find the two points where the object with parameterization $\boldsymbol{r}(t) = t^2\hat{\boldsymbol{\imath}} + t\hat{\boldsymbol{\jmath}} - 30\hat{\boldsymbol{k}}$ hits the plane with equation $x + y + z = 0$. [15 pts]

Solution: