

MATH241 Spring 2023 Exam 1 (Justin W-G)

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Instructions:

1. Please do all problems on the pages and in the spaces provided. This exam will be scanned into Gradescope and if your answers are not in the correct locations they will not be found or graded!
2. Only simplify Calculus 3 related calculations.

1. Write T for True or F for False in the box to the right. No justification is required. Unreadable or ambiguous letters will be marked as incorrect. [10 pts]

Solution:

Statement	T or F
$\bar{\mathbf{a}} \times (\bar{\mathbf{b}} \times \bar{\mathbf{c}})$ is a scalar.	
The planes $2x + 3y - z = 0$ and $-4x - 6y + 2z = 100$ are parallel.	
The spheres $x^2 + y^2 + z^2 = 100$ and $x^2 + y^2 + (z - 20)^2 = 100$ meet.	
Acceleration always has positive magnitude.	
a_N is always non-negative.	

2. Given the vectors:

$$\bar{\mathbf{a}} = 2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$

$$\bar{\mathbf{b}} = 0\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$$

$$\bar{\mathbf{c}} = 3\hat{\mathbf{i}} + \alpha\hat{\mathbf{j}} + \alpha\hat{\mathbf{k}}$$

- (a) Calculate $\text{Proj}_{\bar{\mathbf{b}}}\bar{\mathbf{a}}$.

[10 pts]

Solution:

- (b) Is it possible to choose α so that $\bar{\mathbf{a}}$ and $\bar{\mathbf{c}}$ are parallel? Explain.

[5 pts]

Solution:

3. Find the distance between the point $(1, 2, 3)$ and the plane $5x + y - 10z = 10$. Simplify. [10 pts]

Solution:

4. Find the equation of the plane containing the point $(3, 4, -1)$ and perpendicular to both of the [10 pts]
planes:

$$2x + 4y - z = 0 \quad \text{and} \quad x + 5z = 6$$

Write the result in the form $ax + by + cz = d$.

Solution:

5. Suppose an object follows the path given by the parameterization:

[10 pts]

$$\bar{\mathbf{r}}(t) = t^3\hat{\mathbf{i}} + t\hat{\mathbf{j}} - t\hat{\mathbf{k}}$$

The object intersects each of the planes $x + y + z = 8$ and $y - z = 10$ exactly once. Write down an integral for the distance the object travels between these two intersections. Do not evaluate.

Solution:

6. Assuming $\beta > 0$, for which values of β is the following parameterization smooth? Explain.

[10 pts]

$$\bar{\mathbf{r}}(t) = \sin(\pi t)\hat{\mathbf{i}} + (t^2 - t)\hat{\mathbf{j}} + 5\hat{\mathbf{k}} \quad \text{with } 0 \leq t \leq \beta$$

Solution:

7. Find the symmetric equation of the line through the two points $(1, 4, -5)$ and $(8, 4, 10)$. [10 pts]

Solution:

8. Write down a parameterization of the line segment from $(1, 0, 5)$ to $(6, 3, 5)$. [5 pts]

Solution:

9. Write down a parameterization of the circle $x^2 + z^2 = 9$ in the plane $y = 2$. [5 pts]

Solution:

10. Plot each of the following in 3D. On each, mark at least one point with its coordinates.

(a) $x^2 + (y - 4)^2 + z^2 = 4$

[5 pts]

Solution:

(b) $x + 2y = 10$

[5 pts]

Solution:

(c) $\vec{r}(t) = t\hat{i} + 1\hat{j} + t^2\hat{k}$ for $0 \leq t \leq 3$

[5 pts]

Solution: