1. (a) Use the gradient to find a vector perpendicular to the graph of the curve \( y = x^3 + x - 2 \) at the point where \( x = 2 \).

**Partial Solutions:** Move all the variables to one side and let that side be the function \( f(x, y) \). Then find \( \nabla f \) at \((2, 8)\) where the 8 comes from plugging 2 into \( y = x^3 + x - 2 \).

(b) Suppose the base of a triangle is growing at 2 inches per hour while the height is growing at 3 inches per hour. At what rate is the area growing when the height is 10 inches and the base is 20 inches?

**Partial Solutions:** Since \( A = \frac{1}{2}bh \) we can apply the chain rule since \( b \) and \( h \) are functions of \( t \).

2. (a) Sketch the graph of the surface \( y^2 = x^2 + z^2 \). Write the name.

**Partial Solutions:** Sketch omitted - this is a double-cone opening both directions around the \( y \)-axis with the vertex at the origin.

(b) Sketch the graph of the surface \( y = x^2 \). Write the name.

**Partial Solutions:** Sketch omitted - this is a parabolic sheet - take \( y = x^2 \) in the \( xy \)-plane and extend it in the \( z \)-direction.

(c) Find the directional derivative of \( f(x, y) = y \sin(xy) \) in the direction of \( \vec{a} = 2\hat{i} + \hat{j} \) at the point \( (\pi/8, 2) \). Simplify.

**Partial Solutions:** First make \( \vec{a} \) a unit vector then simply apply the formula for the directional derivative.

3. (a) All together on one graph sketch the level curves for \( f(x, y) = y - |x| \) at \( c = -2, 0, 2 \) and label each with its value of \( c \).

**Partial Solutions:** Sketches omitted - The curves are \( y = |x| - 2, y = |x| \) and \( y = |x| + 2 \). These are all absolute value vertical shifts.

(b) Suppose the unit vector \( \vec{u} \) makes an angle of 30° with the gradient of a function \( f \) at \((1, 2)\) and \( ||\nabla f(1, 2)|| = 3 \). Find \( D_{\vec{u}}f(1, 2) \).

**Partial Solutions:** Remember that \( D_{\vec{u}}f(1, 2) = \vec{u} \cdot \nabla f(1, 2) = ||\vec{u}|| ||\nabla f(1, 2)|| \cos \theta \). Then \( \vec{u} \) is a unit vector and plug the rest in.

(c) The function \( f(x, y) = x^2y - 2x^2 - y^2 \) has the following:

\[
\begin{align*}
  f_{xx}(x, y) &= 2y - 4 \\
  f_{yy}(x, y) &= -2 \\
  f_{xy}(x, y) &= 2x
\end{align*}
\]

There are three critical points at \((0, 0)\), \((2, 2)\) and \((-2, 2)\). Categorize each critical point as a relative maximum, relative minimum or saddle point.

**Partial Solutions:** This is pretty straightforward since everything is given. Don’t get the discriminant incorrect!
4. Find the maximum and minimum values of \( f(x, y) = x^2 + 2y^2 \) on the quarter circle \( x^2 + y^2 \leq 4 \) with \( x, y \geq 0 \).

**Partial Solutions:** The critical point is at \((0, 0)\) which is on the edge and \( f(0, 0) = 0 \). The edge is composed of three pieces:

- The quarter circle \( x^2 + y^2 = 4 \) where \( y^2 = 4 - x^2 \) and so \( f = x^2 + 2(4 - x^2) = 8 - x^2 \) with \( 0 \leq x \leq 2 \). What’s the max and min?
- The lower line segment (on the \( x \)-axis) which is \( y = 0 \) and so \( f = x^2 \) with \( 0 \leq x \leq 2 \). What’s the max and min?
- The left line segment (on the \( y \)-axis) which is \( x = 0 \) and so \( f = 2y^2 \) with \( 0 \leq y \leq 2 \). What’s the max and min?

Pick the max and min from among them all.

5. Let \( f(x, y) = x^2 + 6y^2 \) and suppose \((x, y)\) is constrained by \( x + 3y = 10 \).

(a) Use Lagrange multipliers to find the minimum of \( f(x, y) \) subject to the constraint. [16 pts]

**Partial Solutions:** The system is:

\[
\begin{align*}
2x &= \lambda(1) \\
12y &= \lambda(3) \\
x + 3y &= 10
\end{align*}
\]

This has only one solution at \((4, 2)\).

(b) Explain why \( f(x, y) \) has no maximum subject to the constraint. [4 pts]

**Partial Solutions:** Because on the constraint \( x = 10 - 3y \) and so \( f(x, y) = x^2 + 6y^2 = (10 - 3y)^2 + 6y^2 = 100 - 60y + 15y^2 \) which can be made as large as we want by making \( y \) large.