Math 241 Excellent Exam 2 Fall 2015 **Justin Wyss-Gallifent**

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. (a) Sketch the graph of the function $f(x,y) = 4 - y^2$. Include some sense of size and position. Name the shape. [5 pts]

   (b) Write down the equation for the cone opening around the positive y-axis with vertex at the origin. [5 pts]

   (c) All together on one xy-plane sketch the level curves for $f(x,y) = x - y^2$ for $c = -2, 0, 2$. Label each with its value of c. [10 pts]

Please put problem 2 on answer sheet 2

2. (a) Find the directional derivative of $f(x,y) = \frac{x^2}{y^3}$ at $(2,-1)$ in the direction of $1\,\hat{\imath} - 1\,\hat{\jmath}$. [8 pts]

   (b) Electrical power P (in watts) can be measured as a function of voltage V (in volts) and resistance R (in ohms) by the formula [12 pts]

$$P = \frac{V^2}{R}$$

   If the voltage is decreasing at 2 volts/second while the power is decreasing at 3 watts/second, at what rate is the resistance changing when $V = 10$ and $R = 15$?

Please put problem 3 on answer sheet 3

3. (a) Approximate the value $\sqrt{2.99^2 - 4.97}$ using tangent plane approximation. [10 pts]

   (b) Find the equation of the plane tangent to the surface $x = y^3 - z^2$ at $(-1,2,3)$ [10 pts]

Please put problem 4 on answer sheet 4

4. Let $f(x,y) = x^2y - x^2 - 2y^2$

   (a) Show that the only critical points for this function are $(0,0)$, $(2,1)$ and $(-2,1)$. [8 pts]

   (b) Categorize each critical point as a relative maximum, relative minimum or saddle point. [12 pts]

Please put problem 5 on answer sheet 5

5. Use Lagrange Multipliers to find the maximum and minimum values of $f(x,y) = 2x^2 + \frac{2}{3}y^3$ on the circle $x^2 + y^2 = 4$. [20 pts]

TAs are:
Maxx (0311=10:00, 0321=11:00)
Danul (0312=10:00, 0342=1:00)
Daniel (0322=11:00, 0332=12:00)
Douglas (0331=12:00, 0341=1:00)

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