**MATH241 Exam 2 Fall 2021 Justin Wyss-Gallifent**

| **Your Name NEATLY:** |
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**Directions:** Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

1. Together in space, sketch the graphs of the two surfaces $x^2 + z^2 = 16$ and $y = x^2 + z^2$. Describe the intersection of these two surfaces. Use both words and equations to be precise. [10 pt]

   **Solution:**

2. Write down the equation of the cylinder with axis $x = 1$, $z = 2$ and with radius 3. [5 pt]

   **Solution:**

3. Write down the function $f(x, y) = ...$ for the plane with intercepts $(2, 0, 0)$, $(0, 4, 0)$, and $(0, 0, 5)$. [5 pt]

   **Solution:**

4. All together on one set of axes draw the level curves of $f(x, y) = \frac{y}{|x|+1}$ for the values $c = -2$, 0, and 2. Label each curve with its value of $c$. [10 pt]

**Solution:**

5. Find the equation of the plane tangent to the graph of $f(x, y) = x^2y - y$ at the point where $x = 2$ and $y = -1$. Write it in the form $ax + by + cz = d$. [10 pt]

**Solution:**

6. Suppose the location of an object is given by $\mathbf{r}(t) = (t^2 + t)\hat{\boldsymbol{\imath}} + e^{2t-4}\hat{\boldsymbol{\jmath}}$. Suppose the temperature (in $^\circ$C) of the plane (measured in meters) is given by $f(x, y) = x^2 + xy$

   (a) Determine the instantaneous rate of change of temperature in $^\circ$C/m that the object is experiencing as it passes through the point $(6, 1)$. [15 pt]

   **Solution:**

   (b) At the point $(6, 1)$ in which direction should it go if it wants to experience the maximum instantaneous change of temperature? [5 pt]

   **Solution:**

7. Find and categorize all three critical points (as relative maxima, relative minima, or saddle points) for the function $f(x, y) = 4xy - x^4 - y^4$. [20 pt]

**Solution:**

8. Use the method of Lagrange Multipliers to find the maximum and minimum values of the objective function $f(x, y) = xy$ subject to the constraint $x^2 + (y + 1)^2 = 1$. [20 pt]

**Solution:**