MATH241 Fall 2022 Exam 2 (Justin W-G)

NAME (Neatly):

UID (Neatly):

Instructions:

1. Please do all problems on the pages and in the spaces provided. This exam will be scanned into Gradescope and if your answers are not in the correct locations they will not be found or graded!
2. Only simplify Calculus 3 related calculations.

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(Except for doodling for stress relief.)

1. Together in space, sketch the graphs of the two surfaces $y = \sqrt{x^2 + z^2}$ and $y = 4$. Describe the intersection of these two surfaces. Use both words and equations to be precise. [10 pt]

   Solution:

2. Write down the equation of the paraboloid with vertex at $(5, 0, 0)$ opening towards the negative x-axis. [5 pt]

   Solution:

3. Write down the function $f(x, y) = ...$ for the plane with x-intercept $(2, 0, 0)$, z-intercept $(0, 0, 4)$, and no y-intercept. [5 pt]

   Solution:

4. All together on one set of axes draw the level curves of $f(x, y) = \frac{x}{|y|+1}$ for the values $c = -2$, 0, and 2. Label each curve with its value of c. [10 pt]

Solution:

5. Use tangent plane approximation to find an approximation of: [10 pt]

$$\sin\left(\frac{11\pi}{12}\right)\sqrt{3.9}$$

Simplify as much as possible but don't approximate your final answer.

Solution:

6. Suppose the location of an object is given by $\bar{r}(t) = (t^2 + t)\hat{\boldsymbol{\imath}} + e^{2t-4}\hat{\boldsymbol{\jmath}}$. Suppose the temperature (in °C) of the plane (measured in meters) is given by $f(x, y) = x^2y - y$. At the instant when $t = 2$, is the object traveling in the direction of greatest temperature increase? Justify. [20 pt]

Solution:

7. Find and categorize both (there are two) critical points (as relative maxima, relative minima, or saddle points) for the function: [20 pt]

$$f(x, y) = \frac{1}{12}x^3 + 10xy + 5y^2$$

Solution:

8. Find the absolute maximum and minimum of the function $f(x, y) = xy$ on the filled-in triangle with corners $(2, 0)$, $(0, 2)$, and $(2, 2)$. [20 pt]

Solution: