MATH241 Fall 2023 Exam 2 (Justin W-G)

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Instructions:

1. Please do all problems on the pages and in the spaces provided. This exam will be scanned into Gradescope and if your answers are not in the correct locations they will not be found or graded!
2. Only simplify Calculus 3 related calculations unless otherwise specified.

1. Write TRUE or FALSE in the box to the right. No justification is required. Unreadable or ambiguous answers will be marked as incorrect. [10 pts]

Solution:

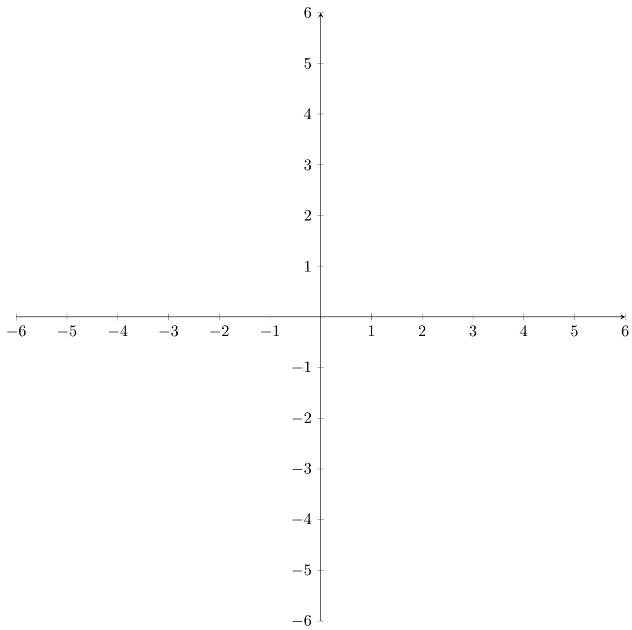
Statement	TRUE/FALSE
$\nabla f(x, y)$ is perpendicular to the plane tangent to $f(x, y)$.	
For $D_{\bar{\boldsymbol{u}}} f$ the vector $\bar{\boldsymbol{u}}$ must be a unit vector.	
Lagrange Multipliers can find a maximum on a filled-in region.	
The discriminant is $D(x, y) = f_{xx} f_{yy} - f_{xy}^2$.	
The equation $x^2 + y^2 + z^2 = 9$ is also a function.	

2. Sketch the graph of the equation $y = -\sqrt{9 - x^2 - z^2}$. Name the shape. [5 pts]

Solution:

3. All together on one xy-plane sketch the level curves for $f(x, y) = x - y^2$ for $c = -2, 0, 2$. Label each with its value of c. [10 pts]

Solution:



4. Suppose $f(x, y) = xy + 3y^2$. If $\bar{\boldsymbol{u}}$ is a unit vector which makes an angle of $\pi/6$ with ∇f at $(2, -1)$, find $D_{\bar{\boldsymbol{u}}} f(2, -1)$. [10 pts]

Solution:

5. Find a vector perpendicular to the graph of $f(x, y) = x\sin(xy)$ at the point $(2, \pi/12)$. Evaluate the trigonometry but don't simplify further. [7 pts]

Solution:

6. The temperature of the plane in $^\circ C$ at (x, y) is $T(x, y) = x\sqrt{y^2 + 9}$. An object is following the path given by the following where t is in seconds: [10 pts]

$$\bar{\boldsymbol{r}}(t) = 3t\,\hat{\boldsymbol{\imath}} + t^2\,\hat{\boldsymbol{\jmath}}$$

What is the rate of change in $^\circ C/s$ that the object is undergoing at the instant when $t = 2$?

Solution:

7. Use tangent plane approximation to approximate the value of $\ln(2x + y)$ at $(x, y) = (-0.9, 3.2)$. [8 pts] Simplify.

Solution:

8. Find all three of the critical points for the function $h(x, y) = x^2y - 2x^2 - y^2$. For each critical point calculate if it is a relative maximum, relative minimum, or saddle point. [20 pts]

Solution:

9. Use the method of Lagrange multipliers to find the maximum and minimum of $f(x, y) = 4xy$ on the circle $x^2 + (y - 1)^2 = 1$. [20 pts]

Solution: