Math 241 Exam 2 Spring 2015

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Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. (a) Sketch the graph of the function \( f(x, y) = 4 - \sqrt{x^2 + y^2} \). Include some sense of size and position. Name the shape. [5 pts]

(b) Write down the equation for the cylinder of radius 5 centered around the \( x \)-axis. [5 pts]

(c) All together on one \( xy \)-plane sketch the level curves for \( f(x, y) = y - 2|x| + 1 \) for \( c = -2, 0, 2 \). Label each with its value of \( c \). [10 pts]

Please put problem 2 on answer sheet 2

2. (a) Find the directional derivative of \( f(x, y) = x^2 y^3 \) at \((1, -2)\) in the direction of \(1 \hat{i} - 1 \hat{j} \). [8 pts]

(b) Electrical power \( P \) (in watts) can be measured as a function of voltage \( V \) (in volts) and resistance \( R \) (in ohms) by the formula

\[
P = \frac{V^2}{R}
\]

If the power is increasing at 5 watts/second while the resistance is decreasing at 2 ohms/second, at what rate is the voltage changing when \( V = 10 \) and \( R = 50 \)? [12 pts]

Please put problem 3 on answer sheet 3

3. (a) Approximate the value \( \sqrt{5.1^2 - 8.9} \) using tangent plane approximation. [10 pts]

(b) Find the parametric equations of the line perpendicular to the surface \( x = y^3 - z^2 \) at \((-1, 2, 3)\). [10 pts]

Please put problem 4 on answer sheet 4

4. Let \( f(x, y) = x^2 y - x^2 - 2y^2 \)

(a) Show that the only critical points for this function are \((0, 0)\), \((2, 1)\) and \((-2, 1)\). [8 pts]

(b) Categorize each critical point as a relative maximum, relative minimum or saddle point. [12 pts]

Please put problem 5 on answer sheet 5

5. Let \( R \) be the triangular region with corners \((2, 0)\), \((2, 2)\) and \((0, 2)\).

(a) Draw a shaded-in picture of \( R \). [5 pts]

(b) Find the maximum and minimum values, as well as where they occur, for the function \( f(x, y) = x^2 + y^2 \) constrained within \( R \). [15 pts]

The End