MATH 241 Sections Exam 2 Spring 2021 (JWG)

Exam Submission:

1. From the moment you download this exam you have three hours to take the exam and submit to Gradescope. This includes the entire upload and tag procedure so do not wait until the last minute.
2. Tag your problems! Please!
3. You may print the exam, write on it, scan and upload.
4. Or you may just write on it on a tablet and upload.
5. Or you are welcome to write the answers on a separate piece of paper if other options don't appeal to you, then scan and upload.

Exam Rules:

1. You may ask for clarification on questions but you may not ask for help on questions!
2. You are permitted to use your notes and the textbook. You are permitted to use a calculator for basic arithmetic.
3. You are not permitted to use other resources. Thus no friends, internet, etc.
4. By taking this exam you agree that if you are found in violation of these rules that the minimum penalty will be a grade of 0 on this exam.

Work Shown:

1. Show all work as appropriate for and using techniques learned in this course.
2. Any pictures, work and scribbles which are legible and relevant will be considered for partial credit.

1. (a) On the same set of axes plot the level curves for $f(x, y) = x + y^2$ for the values $c = -10, 0, 10$ and label each curve with its value of c. Also, label your axes! [10 pts]

(b) Sketch the level surface for $f(x, y, z) = y + 10 - \sqrt{x^2 + z^2}$ for the value $c = 5$. Also, label your axes! [5 pts]

2. Find the equation of the plane tangent to the graph of $f(x, y) = x^2y + x$ at the point where $x = 2$ and $y = 3$. Simplify this to the form $ax + by + cz = d$. [10 pts]

Solution:

3. Suppose for some $f(x, y)$ (not given) and for some unit vector $\bar{\boldsymbol{u}}$ (also not given) you calculate that $D_{\bar{\boldsymbol{u}}} f(3, 4) = 8$ and $\nabla f(3, 4) = 5\,\hat{\boldsymbol{\imath}} + 5\,\hat{\boldsymbol{\jmath}}$. Explain how you know you made a mistake. [5 pts]

Solution:

4. A cylinder is growing such that its radius is changing at 2 cm/sec while its volume is changing at $3\pi\,\text{cm}^3$/sec. At what rate is its height changing at the instant when the height is 100 cm and the volume is $20\pi\,\text{cm}^3$? Include units. [10 pts]

Solution:

5. An object in the plane follows the path $\bar{\boldsymbol{r}}(t) = t^2\,\hat{\boldsymbol{\imath}} + (2t^2 - t)\,\hat{\boldsymbol{\jmath}}$. The temperature at the point (x, y) is given by $f(x, y) = x^2 + x^2y^2$. Assume temperature in $^\circ C$ and distance is in meters and give units.

   (a) At $t = 3$ what is the direction of maximum increase of f? [5 pts]
   Solution:

   (b) At $t = 3$ what instantaneous temperature change is the object experiencing? [10 pts]
   Solution:

6. **Note:** Let G be the largest single digit of your UID. Write down your UID and the value of G and mark them clearly. In the problem below, replace G by the appropriate value before proceeding. [15 pts]

   Consider the function:

$$f(x, y) = x^2y + 2y^2 - (2G + 2)xy + (G^2 + 2G - 15)y$$

   This function has three critical points. Find and classify as either relative max, relative min, or saddle points.

   Solution:

7. **Note:** Let E be the sum of the leftmost three digits of your UID. Write down your UID and the value of E and mark them clearly. In the problem below, replace E by the appropriate value before proceeding. [15 pts]

   Find the maximum of the function:

$$f(x, y) = Exy$$

   Where (x, y) is constrained within the region in the plane between $y = x^2$ and $y = 9$.

   Solution:

8. **Note:** Let F be the sum of the leftmost four digits of your UID. Write down your UID and the value of F and mark them clearly. In the problem below, replace F by the appropriate value before proceeding. [15 pts]

   Use Lagrange Multipliers to find the minimum value of the function:

$$f(x, y) = (x - F)^2 + y^2$$

   Subject to the constraint:

$$x^2 + y^2 = F^2$$

   Solution: