**MATH241 Exam 2 Spring 2022 (Justin Wyss-Gallifent)**

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| **Directions:** Please do work in the spaces provided and do not spill over to other pages - the exams will be scanned into Gradescope and auto-tagged this way. Use only methods taught in this course and show work as indicated. No calculators or other devices permitted. Numerical answers do not need to be simplified. Good luck! |
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1. Draw a reasonable sketch of the equation: [10 pts]

$$y = 4 - \sqrt{x^2 + z^2}$$

**Solution:**

2. All together on one set of 2D axes draw the level curves of $f(x, y) = \frac{x}{y^2}$ for the values $c = -1$, 0, and 1. Label each curve with its value of $c$. [10 pt]

**Solution:**

3. An object is traveling along the line $y = 2x+1$ heading up and to the right. If the temperature at $(x, y)$ in degrees celsius is given by $f(x, y) = x^2y + x - y$, and if the plane is measured in meters, what is the instantaneous temperature change the object is experiencing at the instant when $x = 3$? [15 pts]

**Solution:**

4. Suppose all you know is: [10 pts]

$$z = x\sin(xy)$$
$$x = e^{st}$$
$$y = st$$

Use the Chain rule (from 13.4) to find $\frac{\partial z}{\partial s}$ at $s = 2$ and $t = 1$.

**Solution:**

5. Find the parameterization $\boldsymbol{r}(t) = ...$ of the line perpendicular to the surface $z = x^2 + y$ at the point $(1, 2, 3)$. [10 pts]

**Solution:**

6. Use tangent plane approximation with a nearby reasonable point to approximate the value of: [10 pts]

$$(\ln(e - 0.1))\sqrt{9.2}$$

**Solution:**

7. Find and categorize both (there are two) critical points (as relative maxima, relative minima, or saddle points) for the function: [15 pt]

$$f(x, y) = \frac{1}{12}x^3 + 10xy + 5y^2$$

**Solution:**

8. Use the method of Lagrange Multipliers to find the maximum and minimum values of the objective function $f(x, y) = 2xy$ subject to the constraint $(x - 2)^2 + y^2 = 4$. [20 pt]

**Solution:**