1. (a) Let R be the region inside $r = 1 + \sin\theta$, above $y = \frac{3}{4}$ and in the first quadrant. Draw a picture of R. Set up an iterated integral in polar coordinates for $\iint_R x^2 y \, dA$. [10 pts]

   Partial Solutions: Pictures omitted. The line $y = \frac{3}{4}$ in polar is $r = \frac{3}{4}\csc\theta$ which meets $r = 1 + \sin\theta$ (which is a cardiod) in the first quadrant at $\theta = \frac{\pi}{6}$. To see this set them equal and solve the icky quadratic which results. The final answer is

$$\int_{\pi/6}^{\pi/2} \int_{\frac{3}{4}\csc\theta}^{1+\sin\theta} (r\cos\theta)^2 (r\sin\theta) r \; dr \; d\theta$$

   (b) Let D be the solid in the first octant, inside the sphere $x^2 + y^2 + z^2 = 9$ and below the cone $z = \sqrt{3x^2 + 3y^2}$. Set up an iterated integral in spherical coordinates for $\iiint_D z \, dV$. [10 pts]

   Partial Solutions: Note that the solid is below the cone, which means outside. The cone is $\phi = \frac{\pi}{6}$. The final answer is

$$\int_0^{\pi/2} \int_{\pi/6}^{\pi/2} \int_0^3 \rho\cos\phi \rho^2 \sin\phi \; d\rho \; d\phi \; d\theta$$

2. (a) Let R be the region in the first quadrant bounded by the curve $y = 9 - x^2$. Draw a picture of R and then set up an iterated integral for $\iint_R x \, dA$ with R as HS. [10 pts]

   Partial Solutions: Pictures omitted. As HS in the first quadrant we need to rewrite $y = 9 - x^2$ as $x = +\sqrt{9-y}$. The final answer is

$$\int_0^9 \int_0^{\sqrt{9-y}} x \; dx \; dy$$

   (b) Let R be the region in the second quadrant and bounded by the line $y = x + 3$. Draw a picture of R and then set up an iterated integral for $\iint_R x^2 + y^2 \, dA$ with R as polar. [10 pts]

   Partial Solutions: Pictures omitted. In polar the line is $r\sin\theta = r\cos\theta + 3$ or $r = \frac{3}{\sin\theta - \cos\theta}$. The final answer is

$$\int_{\pi/2}^{\pi} \int_0^{3/(\sin\theta - \cos\theta)} r^3 \; dr \; d\theta$$

3. (a) SKIP - From 14.9.

   (b) SKIP - From 14.9.

   (c) Evaluate the following integral. [10 pts]

$$\int_0^2 \int_x^2 \sin(y^2) \; dy \; dx$$

   Partial Solutions: We need to reparametrize as HS. The result of reparametrizing is

$$\int_0^2 \int_0^y \sin(y^2) \; dx \; dy$$

   which can be integrated fairly easily.

4. (a) Let D be the solid inside the cylinder $r = \sin\theta$, above $z = 0$ and below $z = 9 - x^2 - y^2$. [10 pts]
Draw separate pictures of D and its corresponding R and then set up an iterated integral in cylindrical coordinates for $\iiint\limits_D y\ dV$.

Partial Solutions: Pictures omitted. The final answer is

$$\int_0^{\pi} \int_0^{\sin\theta} \int_0^{9-r^2} r^2 \sin\theta\ dz\ dr\ d\theta$$

(b) Evaluate the integral [10 pts]

$$\int_0^1 \int_x^0 \int_0^{x+y} y\ dz\ dy\ dx$$

Partial Solutions: Straightforward integral.

5. Perform a change of variables to rewrite the integral $\iint\limits_R x\ dA$, where R is the region bounded by the lines $y = \frac{1}{x}$, $y = \frac{2}{x}$, $y = x$ and $y = 4x$. Draw a picture of R, your new region S and write the new integral as an iterated double integral in u and v. [20 pts]

Partial Solutions: Pictures omitted. Rewrite the curves as $xy = 1$, $xy = 2$, $\frac{y}{x} = 1$ and $\frac{y}{x} = 4$. Set $u = xy$ and $v = \frac{y}{x}$. The Jacobian is $J(x, y) = \frac{1}{2v}$ which you can either get with the shortcut $J(x, y) = 1 \div J(u, v)$ or by solving for $x =$ and $y =$ and finding $J(x, y)$ directly.

In any case we need x so we observe $v = \frac{y}{x}$ so that $y = xv$ and then $u = xy = x(xv) = x^2 v$ so finally $x = \sqrt{u/v}$.

The intermediate answer is:

$$\iint_S \sqrt{u/v} \left|\frac{1}{2v}\right|\ dA$$

The final answer is:

$$\int_1^2 \int_1^4 \sqrt{u/v} \left|\frac{1}{2v}\right|\ dv\ du$$