Your Name NEATLY:

MATH241 Exam 3 Fall 2021 (Justin Wyss-Gallifent)

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

1. The following integral can be evaluated without integrating but by figuring out the volume of the shape. Describe the shape and give the numerical value of the integral. [5 pts]

$$\int_{-2}^{2}\int_{1}^{7}\sqrt{4-x^2}\,dy\,dx$$

Solution:

2. Evaluate the following iterated double integral: [10 pts]

$$\int_{0}^{1}\int_{0}^{y}4x-y\,dx\,dy$$

You Should Evaluate This Integral!

Solution:

3. Let R be the 2D region between the graphs of $y = 5 - x^2$ and $y = 2x$. Write down the iterated double integral for $\iint_R xy\,dA$, treating R as vertically simple. [10 pts]

You Should Not Evaluate Your Resulting Integral!

Solution:

4. Write down a parametrization $\mathbf{r}(?,?) = ...$ of the part of the cylinder $x^2 + y^2 = 9$ between $z = 1$ and $z = 3$ and which is NOT in the first octant. [5 pts]

Solution:

5. Let D be the 3D solid object in the first octant and bounded by the planes $y = 4$, $y = x$, $y = 2x$, and $x + y + z = 10$. Write down an iterated triple integral in rectangular coordinates for the volume of D. [15 pts]

You Should Not Evaluate Your Resulting Integral!

Solution:

6. Let D be the 3D solid object above the xy-plane, inside the cylinder $x^2+y^2=4$ and below/outside the cone $z=\sqrt{\frac{x^2+y^2}{3}}$. If the density of D at the point (x,y,z) equals x^2z, write down the iterated triple integral in spherical coordinates for the mass of D. [20 pts]

You Should Not Evaluate Your Resulting Integral!

Solution:

7. Convert the following integral to an iterated double integral in polar coordinates. [15 pts]

$$\int_1^2 \int_{-\sqrt{1-(x-1)^2}}^{+\sqrt{1-(x-1)^2}} x \, dy \, dx$$

You Should Not Evaluate Your Resulting Integral!

Solution:

8. Let R be the 2D region bounded by the three lines: [20 pts]

$$y = x + 1$$

$$y = 3x$$

$$y = 4x$$

Use the change of variables $x = u + v$ and $y = u + 4v$ to convert the following integral to a double iterated integral in the uv-plane.

$$\iint_R y \, dA$$

You Should Not Evaluate Your Resulting Integral!

Solution: