MATH241 Fall 2022 Exam 3 (Justin W-G)

NAME (Neatly):

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Instructions:

1. Please do all problems on the pages and in the spaces provided. This exam will be scanned into Gradescope and if your answers are not in the correct locations they will not be found or graded!
2. Only simplify Calculus 3 related calculations.

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(Except for doodling for stress relief.)

1. The following integral can be evaluated without integrating but by understanding something about the shape and integrand. Describe the shape and give the numerical value for the integral. [10 pts]

$$\int_0^{\pi} \int_0^{3} \int_0^{\sqrt{9-r^2}} r \, dz \, dr \, d\theta$$

Solution:

2. Given the following iterated integral:

$$\int_0^1 \int_{\sqrt{y}}^1 3\,dx\,dy$$

(a) Draw the region R corresponding to the following iterated integral. All you need to do is draw the region. [5 pts]

Solution:

(b) Evaluate the integral. [10 pts]

Solution:

3. Suppose R is the filled-in triangle with corners $(0, 0)$, $(2, 2)$, and $(2, 4)$. Set up the iterated double integral in rectangular coordinates for: [10 pts]

$$\iint_R xy \, dA$$

DO NOT EVALUATE!

Solution:

4. Suppose R is the region inside $r = \cos\theta$ and outside $r = \sqrt{3}\sin\theta$. Set up the iterated double integral(s) in polar coordinates for: [10 pts]

$$\iint_R x - y \, dA$$

DO NOT EVALUATE!

Solution:

5. Suppose D is the solid under $z = 9 - x^2$, above the xy-plane, and between $y = 1$ and $y = 4$. Write down the iterated triple integral in rectangular coordinates for the volume of D. [15 pts]

DO NOT EVALUATE!

Solution:

6. Suppose D is the solid inside the cone $z = \sqrt{x^2 + y^2}$ and between the planes $z = 2$ and $z = 7$. If the density is given by $f(x, y, z) = xz$ write down the iterated triple integral in spherical coordinates for the mass of D. [15 pts]

DO NOT EVALUATE!

Solution:

7. Write down a parameterization of the part of the paraboloid $z = 9 - x^2 - y^2$ above the xy-plane. [5 pts] No sketch is required.

Solution:

8. Sketch the surface parameterized by: [5 pts]

$$\boldsymbol{r}(x, z) = x\hat{\boldsymbol{\imath}} + (4 - x)\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}}$$
$$0 \leq x \leq 4$$
$$0 \leq z \leq 7$$

Solution:

9. Let R be the region bounded by the lines $y = 1$, $y = \frac{1}{4}x$, and $x - 3y = 2$. Consider the integral: [15 pts]

$$\iint_R \frac{y}{x - 3y}\, dA$$

Use the substitution $x = 3u + v$ and $y = u$ to convert this integral to an iterated integral in the uv-plane.

DO NOT EVALUATE!

Solution: