Math 241 Exam 3 Spring 2019

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Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. Let \( R \) be the region bounded by \( x = 5 - y^2 \) and \( x = 1 \). Evaluate the following integral. \([20 \text{ pts}]\)

This is the only integral you need to evaluate. You do not need to simplify.

\[
\iint_{R} 1 \, dA
\]

Please put problem 2 on answer sheet 2

2. (a) Let \( R \) be the region inside the cardioid \( r = 2 + 2 \sin \theta \) and outside the circle \( r = 3 \). Write down the double integral in polar coordinates for \( \iint_{R} xy \, dA \). Do not evaluate. \([12 \text{ pts}]\)

(b) Let \( \Sigma \) be the portion of the parabolic sheet \( y = 4 - x^2 \) inside the cylinder \( x^2 + z^2 = 1 \). Write down a parametrization for \( \Sigma \). \([8 \text{ pts}]\)

Please put problem 3 on answer sheet 3

3. Reparametrize the following integral as polar. Do not evaluate. \([20 \text{ pts}]\)

\[
\int_{0}^{2} \int_{x}^{4-x} y \, dy \, dx
\]

Please put problem 4 on answer sheet 4

4. (a) Let \( D \) be the solid inside the cylinder \( x^2 + y^2 = 9 \), below the cone \( z = \sqrt{3x^2 + 3y^2} \) and above the \( xy \)-plane. If the density at a point is given by \( f(x, y, z) = x^2 \), set up the iterated integral in cylindrical coordinates for the mass of \( D \). Do not evaluate. \([10 \text{ pts}]\)

(b) Let \( D \) be the solid inside the sphere \( x^2 + y^2 + z^2 = 16 \) and outside the cylinder \( x^2 + y^2 = 4 \). Set up the iterated integral in spherical coordinates for \( \iiint_{D} z^2 \, dV \). Do not evaluate. \([10 \text{ pts}]\)

Please put problem 5 on answer sheet 5

5. Let \( R \) be the region bounded by the lines \( y = 2x + 1 \), \( y = 2x + 9 \), \( y = 9 - x \), and \( y = 12 - x \). \([20 \text{ pts}]\)

Use a change of variables to convert the integral

\[
\iint_{R} \frac{y - 2x}{y + x} \, dA
\]

into a double integral over a rectangular region. Do not evaluate.

The End and the TA Section List

Chenzhi 0311 ↔ 8:00 0321 ↔ 9:30
Corry 0312 ↔ 8:00 0322 ↔ 9:30
Noah 0331 ↔ 11:00 0341 ↔ 12:30
Papia 0332 ↔ 11:00 0342 ↔ 12:30