**Math 241 Exam 3 Spring 2020** **Justin Wyss-Gallifent**

**Directions:** You may use any resources except asking other people. Answers should be neat and tidy and complete.

1. Consider the integrals: [25pts]

$$A = \int_0^1 \int_0^x x^2 y \, dy \, dx \qquad \text{and} \qquad B = \int_0^2 \int_0^x x^2 y \, dy \, dx$$

   (a) Either $A < B$, $A = B$ or $A > B$. Without calculating either integral explain in a few sentences which is true and why. You may use pictures too if you feel it helps.

   (b) Calculate both integrals. Were you correct?

2. Let $R$ be the region inside $r = 2\cos\theta$ and to the right of $x = 1$. Consider the integral: [25pts]

$$\iint_R x \, dA$$

   (a) Draw a picture of $R$.

   (b) Parametrize $R$ as polar and write down the corresponding iterated integral. Do not evaluate.

   (c) Parametrize $R$ as vertically simple and write down the corresponding iterated integral. Do not evaluate.

   (d) Parametrize $R$ as horizontally simple and write down the corresponding iterated integral. Do not evaluate.

3. Let $D$ be the solid above the cone $\phi = \phi_0$ and inside the sphere of radius $\rho = \rho_0$. Here both $\phi_0$ and $\rho_0$ are unknown constants. [25pts]

   (a) Use a triple integral in spherical coordinates to find a formula for the volume of $D$. Your answer will have $\phi_0$ and $\rho_0$ in it.

   (b) When $\phi_0 = \pi/2$ and $\rho_0 = 3$ what does $D$ look like? When you plug these into your formula do you get the answer you expect? Explain in a sentence or two.

   (c) When $\phi_0 = \pi$ and $\rho_0 = 5$ what does $D$ look like? When you plug these into your formula do you get the answer you expect? Explain in a sentence or two.

   (d) Explain in a few sentences why cylindrical coordinates would be a really difficult way to do part (a).

4. Let $R$ be the region bounded by the lines $y = x$, $y = x - 4$, $y = -x$ and $y = -x + 4$. Consider the integral: [25pts]

$$\iint_R x \, dA$$

   (a) Parametrize $R$ using two vertically simple regions and evaluate.

   (b) Use a change of variables to rewrite $R$ as a square in the $uv$-plane and evaluate.

   (c) These values should be the same. Are they?