Math 241 Ephemeral Exam 4 Fall 2015 **Justin Wyss-Gallifent**

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. (a) Evaluate $\int_C 8y\ ds$ where C is parametrized by $\mathbf{r}(t) = t^2\,\mathbf{i} + t\,\mathbf{j}$ for $0 \leq t \leq 3$. [10 pts]

   (b) Use the Fundamental Theorem of Line Integrals to evaluate $\int_C \left(1\,\mathbf{i} + \frac{1}{z}\,\mathbf{j} - \frac{y}{z^2}\,\mathbf{k}\right) \cdot d\mathbf{r}$ where C is parametrized by $\mathbf{r}(t) = t^2\,\mathbf{i} + t\sin(\pi t)\,\mathbf{j} + \frac{1}{t}\,\mathbf{k}$ for $\frac{1}{2} \leq t \leq 1$. [10 pts]

   Extra Credit: At the bottom of the first sheet put the date, time, building (code or full name) and room number of your final exam. [+3 pts]

Please put problem 2 on answer sheet 2

2. Set up the integral for the mass of Σ, the part of the cylinder $x^2 + z^2 = 4$ between $y = 0$ and $y = 3$ if the mass density at (x, y, z) is given by $\delta(x, y, z) = x^2$. Proceed until you have an iterated double integral but do not evaluate. [20 pts]

Please put problem 3 on answer sheet 3

3. Let C be the triangle with corners $(-6, 0)$, $(0, 3)$ and $(3, 0)$ with clockwise orientation. Use Green's Theorem to evaluate $\int_C y^2\ dx + 6xy\ dy$. [20 pts]

Please put problem 4 on answer sheet 4

4. Let C be the intersection of the parabolic sheet $z = 9 - x^2$ with cylinder $r = \sin\theta$, oriented counterclockwise when viewed from above. Apply Stokes' Theorem to the line integral [20 pts]

$$\int_C (xz\,\mathbf{i} + x\,\mathbf{j} + y\,\mathbf{k}) \cdot d\mathbf{r}$$

   Parametrize the resulting surface and proceed until you have an iterated double integral but do not evaluate.

Please put problem 5 on answer sheet 5

5. (a) Let $f(x, y) = x^2 y$. Sketch the vectors for the vector field ∇f at the points $(1, 2)$ and $(2, 1)$. [5 pts]

   (b) Evaluate $\iint_\Sigma (x\,\mathbf{i} + xz\,\mathbf{j} + 5z\,\mathbf{k}) \cdot \mathbf{n}\ dS$ where Σ is the part of the sphere $x^2 + y^2 + z^2 = 9$ above the xy-plane along with the disk $x^2 + y^2 \leq 9$ in the xy-plane. Assume Σ has outwards orientation. [15 pts]

Maxx (0311=10:00, 0321=11:00)
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Daniel (0322=11:00, 0332=12:00)
Douglas (0331=12:00, 0341=1:00)

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