Math 241 Σxam 4 Fall 2017 **$\int$ustin $\nabla\nabla$yss-Gallifent**

Directions: Do not simplify, evaluate or integrate unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words, pictures or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. (a) Let C be the line segment from $(0,0)$ to $(3,2)$. Evaluate $\int_C x+y\,ds$. [10 pts]

   (b) Evaluate [10 pts]

$$\int_C (y+2xz)\,dx + x\,dy + x^2\,dz$$

   where C is the curve parametrized by

$$\mathbf{r}(t) = (2t^2+t)\,\mathbf{i} + (3-t)\,\mathbf{j} + 5\sqrt{t}\,\mathbf{k} \text{ for } 1 \le t \le 4$$

   Extra Credit: At the bottom of the first sheet put the date, time, building (code or full name) and room number of your final exam. [+3 pts]

Please put problem 2 on answer sheet 2

2. (a) Apply Green's Theorem to $\int_C x^7\,dx + 3x^2\,dy$ where C is the cardioid $r = 1+\cos\theta$ with clockwise orientation. Proceed until you have an iterated double integral. [10 pts]
   Do Not Evaluate This Integral.

   (b) Let $\mathbf{F}(x,y,z) = z\,\mathbf{i} + xy\,\mathbf{j} + xy\,\mathbf{k}$. Only one of $\nabla \times (\nabla \cdot \mathbf{F})$ and $\nabla \cdot (\nabla \times \mathbf{F})$ makes sense. Calculate the one that does. [10 pts]

Please put problem 3 on answer sheet 3

3. Let Σ be the part of the cylinder $y^2+z^2=4$ between $x=0$ and $x=5$. If the density at any point is $f(x,y,z)=x^2$, write down and evaluate the integral to find the mass of Σ. [20 pts]

Please put problem 4 on answer sheet 4

4. Let Σ be the part of the parabolic sheet $y = 9-x^2$ having $y \ge 0$ and between $z=0$ and $z=5$. Let C be the edge of Σ with counterclockwise orientation when viewed from the positive y-axis. Apply Stokes' Theorem to the line integral [20 pts]

$$\int_C (2\,\mathbf{i} + xy\,\mathbf{j} + xyz\,\mathbf{k}) \cdot d\mathbf{r}$$

   Parametrize the resulting surface and proceed until you have an iterated double integral.
   Do Not Evaluate This Integral.

Please put problem 5 on answer sheet 5

5. Suppose Σ is the portion of the cylinder $x^2+y^2=9$ between $z=2$ and $z=6$ along with the disks which seal it off at each end. Assume Σ is oriented inwards. Use the Divergence Theorem to evaluate the integral [20 pts]

$$\iint_\Sigma (z\,\mathbf{i} + yz\,\mathbf{j} + y\,\mathbf{k}) \cdot \mathbf{n}\; dS$$

The End and the TA Section List

Tessa	0411 = 8:00	0421 = 9:00
Weikun	0412 = 8:00	0422 = 9:00
Shuo	0431 = 10:00	0441 = 11:00
Zeyad	0432 = 10:00	0442 = 11:00