Math 241 Exam 4 Fall 2018 **Justin Wyss-Gallifent**

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.
Note: Several of these problems ask you to evaluate integrals. These are all quite easy and/or use Theorems.

Please put problem 1 on answer sheet 1

1. (a) Let $f(x, y, z) = x^2 + yz^3$. Only one of $\nabla \times (\nabla f)$ and $\nabla \times (\nabla \cdot f)$ makes sense. Calculate the one that does. [10 pts]

   (b) Evalute $\int_C x + y\, ds$ where C is the part of the semicircle $x^2 + y^2 = 9$ above the x-axis. [10 pts]

   Extra Credit: At the bottom of the first sheet put the date, time, building (code or full name) and room number of your final exam. [+3 pts]

Please put problem 2 on answer sheet 2

2. (a) Evaluate $\int_C (2xy + y)\, dx + (x^2 + x)\, dy$ where C is parametrized by $\mathbf{r}(t) = t^2 e^t\,\mathbf{i} + \sqrt{t}\,\mathbf{j}$ for $0 \leq t \leq 4$. [10 pts]

   (b) Evaluate $\int_C (x\,\mathbf{i} + x\,\mathbf{j}) \cdot d\mathbf{r}$ where C is the line segment from $(0, 0)$ to $(4, 2)$. [10 pts]

Please put problem 3 on answer sheet 3

3. Evaluate $\int_C 3y\, dx + x^2\, dy$ where C is the triangle with vertices $(0, 0)$, $(0, 4)$ and $(2, 4)$, oriented clockwise. [20 pts]

Please put problem 4 on answer sheet 4

4. Let Σ be the part of the cylinder $x^2 + y^2 = 4$ in the first octant and below $z = 4$. Let C be the edge of Σ with counterclockwise orientation when viewed looking toward the origin. Apply Stokes' Theorem to the integral $\int_C x^2 z\, dx + y\, dy + xy^2\, dz$ and proceed until you have an iterated double integral. **Do Not Evaluate.** [20 pts]

Please put problem 5 on answer sheet 5

5. (a) Let Σ be the part of the paraboloid $z = x^2 + y^2$ constrained by $0 \leq x \leq 1$ and $0 \leq y \leq 2$. Write down an iterated double integral for the surface area of Σ. **Do Not Evaluate.** [10 pts]

   (b) Apply the Divergence Theorem to $\iint_\Sigma \left(y\,\mathbf{i} + xy^2\,\mathbf{j} + 3z\,\mathbf{k}\right) \cdot \mathbf{n}\, dS$ where Σ is part of the cylinder $x^2 + y^2 = 9$ between $z = 0$ and $z = 4$ with the disks which seal it off at the ends, oriented outwards. Proceed until you have an iterated integral. **Do Not Evaluate.** [10 pts]

The End and the TA Section List

Avi	0311 $\leftrightarrow$ 10:00	0321 $\leftrightarrow$ 11:00
Zeynep	0312 $\leftrightarrow$ 10:00	0322 $\leftrightarrow$ 11:00
Jialin	0331 $\leftrightarrow$ 12:00	0341 $\leftrightarrow$ 1:00
Zack	0332 $\leftrightarrow$ 12:00	0342 $\leftrightarrow$ 1:00